Iconic Diagrams and Dimensional Analysis for Bondgraphs

Prof. Jean Thoma
Department of Systems Design
University of Waterloo, Ont.,
N2L3G1, Canada
http://www.jthoma.ch
jean@jthoma.ch

Dr. Gianni Mocellin
Straco consulting
15, rue du Diorama
1204-Geneva, Switzerland
http://www.straco.ch
mocellin@straco.ch

KEYWORDS
bond graphs, iconic diagrams, dimensional analysis, physical quantities

ABSTRACT

This paper consists of two parts. Firstly, we consider icons instead of BG symbols, which are better accepted by some people. Secondly, we take dimensional analysis for any physical quantities that are built from fundamental and derived quantities. The main thing is that they have a special mathematical structure and only quantities of the same dimension can be equated, added or subtracted. By multiplication, on the other hand, physical quantities can be combined, i.e. area is the product of two lengths and velocity is the quotient of a length and atime.

We develop this algebra where nondimensional combinations are important such as for instance the Reynolds number. This leads to Buchingham's ρ theorem. Also, physical quantities are built from fundamental units such as length, time and mass. However it is clearer to use length, time and force or even length, time and energy instead.

INTRODUCTION

BGs (bondgraphs) are an universal graphical means to represent dynamic systems that we have used for many years: components of systems are represented by letters, that is an alphanumeric code which represents a certain abstraction. Some people prefer a iconic code, which is more suitable for their intuition. So one purpose of this paper is to develop an iconic or picture code to complement BGs.

The other purpose of the paper is dimensional analysis, a very old technique that has spawned a large literature including some papers by Thoma [THOMA 1968]. If done properly, dimensional analysis is a very powerful tool for modelling, especially to set up practical formulae. Let us note also that on all graphical representations the dimensions of the quantities should be given, for instance volt per meter V/m for electric field strength. Another example is the product of voltage and current VA, which is power, expressed in Watt, the time derivative of transported energy. In mechanics we have the newton N and the velocity in meters per second m/s, whose product is again the transported power or time derivative of energy.

Proceedings 20th European Conference on Modelling and Simulation
Wolfgang Borutzky, Alessandra Orsoni, Richard Zobel © ECMS, 2006
ISBN 0-9553018-0-7 / ISBN 0-9553018-1-5 (CD)

ICONS FOR HOT GAS SYSTEMS

According to the general philosophy of hot gas systems, which is developed in [THOMA 2006], thermofluid machines are built quite naturally as a sequence of RECO, HEXA and TEFMA.

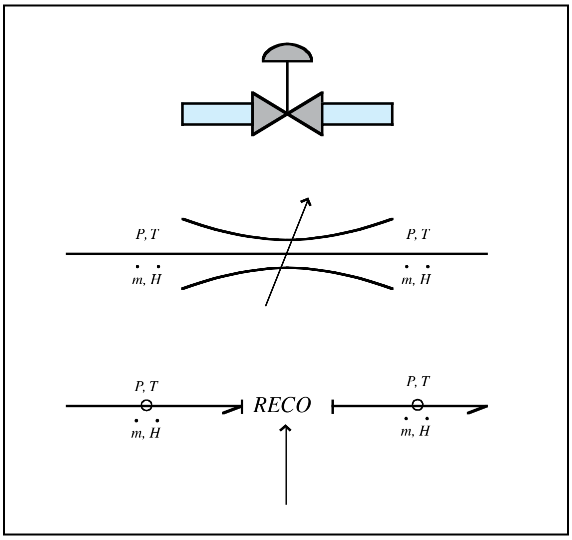

Fig 1 Resistor for hot gas, center as hydraulic symbol and below as BG.

On Fig. 1, we see a picture of a valve on top, a RECO icon in the center and a BG symbol below. RECO is a contraction of 'resistor for convection' and both mass flow and enthalpy flux are conserved in it. The icon comes from hydraulics practice, where it simply means a narrow passage along a fluid line. Let us insist on the fact that mass flow and enthalpy flux are conserved in a RECO.

Next is the heat exchanger which we call HEXA and which appears on Fig. 2.

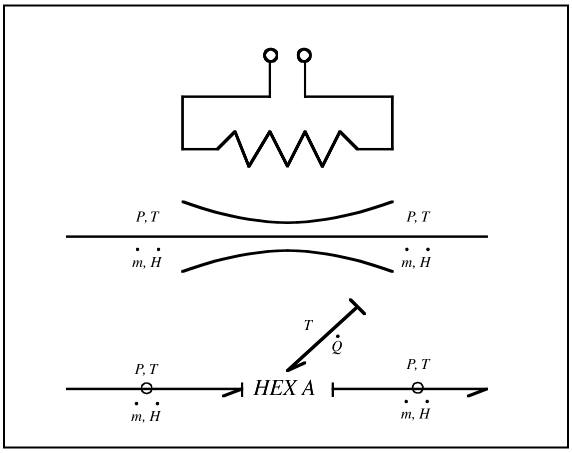

Fig 2 Heat exchanger as an icon above and as BG below. Here mass flow is conserved and enthalpy flux obtained by a simple power balance.

Here we use simply a power balance to obtain the outgoing enthalpy flux:

$$\dot{H}_2 = \dot{H}_1 + \dot{Q}$$

Note that the HEXA needs the following causality: heat flux in and temperature out; this is a consequence of the equation between enthalpy flux and heat flow. The thermal bond is pseudo with temperature as effort variable.

One step further is the thermal fluid machine TEFMA, obtained from the HEXA by adding a mechanical bond; icon and BG appear on Fig 3.

The power balance becomes

$$\dot{H}_2 = \dot{H}_1 + \dot{Q} + M\omega$$

Note that a mechanical bond, which is a true bond, has been added to the HEXA. Causalities are mandatory but they can be turned around by suitable one-ports: the inertia of the rotor and the thermal resistance of the heater.

Some people like the iconic symbols better than BG symbols, although additional information, such as the signs for power for example, must be given as well, especially in electrical circuits.

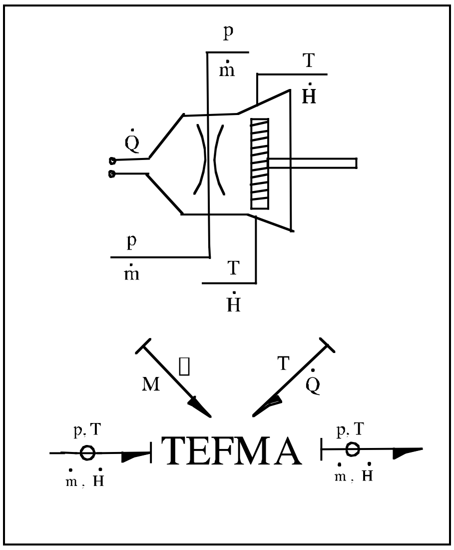

Fig 3 Thermal turbomachine with one additional mechanical bond compared to HEXA, top as icon and below as BG.

In our experience, BG symbols are much easier to handle and we include icons only for illustration.

DIMENSIONAL ANALYSIS

Dimensional analysis is a very powerful tool for the study of systems if done properly [BRIDGMANN 1922, SEDOV 1972, KASPRZAK 1990]. It starts from the observation that all physical quantities (or variables) are the product of a measure q_m and a unit q_u

$$q = q_m \cdot q_u$$

or, in words, the length of something is 10 times the unit of 1 meter.

$$Length = 10 \text{ x } 1\ meter$$

The measure is a (pure) number and the unit is something totally different: it is a quantity associated with the number 1 and has the following properties:

1. Units can be transformed as follows:

$$q = q_{m1} \cdot q_{u1} = q_{m2} \cdot q_{u2}$$

For example :

$$q = 10m = 30.2 feet$$

which means that if the unit decreases, the measure increases, such as to leave the quantity q invariant.

So, we can write equations not only between numbers but also between quantities of the same kind or dimension (in the above example, length).

2. Equations between quantities of different kinds or dimensions can be obtained by multiplication and division:

$$q = q_u \cdot q_u$$

as, for example

$$A = l \,.\, l$$

So, the unit of area A should be chosen as the product of two units of length, which is by no means always done, as in the following counter example: 1 acre is not the product of one foot times one foot.

In particular, instead of a product one can have a division or even a combination of multiplications and divisions:

$$q = q_{m3} \cdot q_{u3} = \frac{q_{m1} \cdot q_{u1}}{q_{m2} \cdot q_{u2}}$$

as, for example:

$$v = \frac{l}{t}$$

or, in words, velocity equals length divided by time.

In mathematics, these products or divisions are called monomials and the better known polynomials are simply an addition of monomials.

So, in dimensional analysis we have to distinguish between the physical quantities, which have a measure and a unit and the measures which are numbers (sometimes called pure numbers).

Numbers can be handled by ordinary mathematics whilst physical quantities can only be combined by multiplication and division according to the above equations of dimensional analysis. Particularly important are the dimensionless quantities that are obtained by multiplication and division of physical quantities where the units cancel.

One example of a dimensionless monomial is the time constant of control engineering

$$\frac{t}{\tau} = \frac{t}{RC}$$

where RC is the so-called characteristic time.

Another example is the Reynolds number which is the ratio of inertia to viscous forces in fluid mechanics :

$$\mathrm{Re} = \frac{l v \rho}{\mu}$$

where l stands for length, v for velocity, ρ for mass density and μ for dynamic viscosity.

The Reynolds number governs the change from laminar to turbulent flow.

All these considerations lead to the ρ theorem of Buckingham [FRENCH 2002] which says that if we have n equations built from m basic units, then they can be reduced to a relation between n-m non-dimensional monomials.

As an example, if we had 5 quantities built from 3 units, we have a relation between two monomials. This means that one of the monomials is a function of the other one. In fluid dynamics, for example, the drag force of a submerged body is the product of dynamic pressure and cross section, which is a function of the Reynolds number.

The difference between physical quantities and measures is important mainly for block diagrams BD and bond graphs BG. In both we may have several control loops which must be dimensionless as they go into a mixing point (addition or subtraction point) to be compared. On the other hand, the gains of simple elements generally have dimensions. So, the loops are dimensionless and the number that represents them can be considered as large or small compared to one.

One example is a servo-amplifier which takes voltage as input and outputs a fluid volume flow. Another example would be a DC electric motor with torque output proportional to input current: the back EMF is proportional to angular velocity. Of course, in BG language, it is a gyrator.

PRISONERS OF MLT UNITS

Theory and practice show that very often one should use force and not mass as a fundamental unit. Mass as unit was choosen only because it is easier to obtain a precise standard of mass than to obtain a standard of force. The use of force as fundamental unit would be somewhat of a return to the old system of technical units. As we like to formulate, one should not be a prisoner of the mass unit because acceleration is only one effect in physics.

Better still is energy as a fundamental unit instead of force, because as force times displacement, energy is a conserved quantity. Very useful also is to consider impulse (momentum) as a fundamental unit because it is also conserved, which is important in modern physics. Force is then simply the time derivative of impulse. So, one can write a BG with impulse derivative as flow, which is then the so-called inverse or force-flow analogy. It brings advantages in understanding certain equilibrium problems of the mechanics of structures, as in civil engineering for example. Here the summing of impulse flows or forces is done in a parallel or 0-junction.

Another quantity to consider as fundamental unit is rotary impulse (angular momentum) which is simply the time integral of torque. It plays an important role in modern micro physics where it is called spin.

These problems can be treated with Systar (*http://www.systar.ch*), a scientific software package in active development [MOCELLIN 2006].
We conclude our overview of icons and dimensional analysis for BG thus: both should remind the reader that BG are not only a mathematical and data processing tool, but also a way of life.

BIBLIOGRAPHY

[BRIDGMANN 1922]
Dimensional Analysis
A. Bridgmann
New Haven, Yale, 1922

[FRENCH 2002]
Conceptual Design for Engineers
M. French
Springer, Heidelberg, 2002

[KASPRZAK 1990]
Dimensional Analysis in the Identification of Mathematical Models
W. Kasprzak, B. Lysik and M, Tybaczuk
World Scientific, Singapore, 1990

[MOCELLIN 2006]
Virtual Reality: the Need for Bond Graphs
G. Mocellin and J. Thoma
European Conference on Modelling and Simulation, Sankt Augustin near Bonn, 2006

[SEDOV 1972]
Similitude et dimensions en mécanique
L. Sedov
Editions Mir, Moscou, 1972

[THOMA 1968]
Physical Quantities and Dimensional Analysis
J. Thoma
Bull. Mech. Eng. Educ
Pergamon Press, Vol. 7, pp 213-226, 1968

[THOMA 2006]
Simulation with Entropy – Understanding Matter and Systems with Bond Graphs
J. Thoma and G. Mocellin
Springer, Heidelberg, 2006