Virtual Reality: the Need for Bond Graphs

Dr. Gianni Mocellin
Straco consulting
15, rue du Diorama
1204-Geneva, Switzerland
http://www.straco.ch
mocellin@straco.ch

Prof. Jean Thoma
Department of Systems Design
University of Waterloo, Ont.,
N2L3G1, Canada
http://www.jthoma.ch
jean@jthoma.ch

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ABSTRACT

In this paper we endeavour to show how the need for modelling and simulation in virtual reality is connected with bond graphs (BG). In effect, both try to immerse their users in realistic virtual worlds, as in flight simulators.

INTRODUCTION

Virtual reality is a branch of computer science that tries to represent dynamic realities as virtual worlds in which the user feels immersed [MOCELLIN 2001].

We show how this science could benefit from bond graphs and how the concepts of modelling and simulation, which are the milestones of bondgraphing, are connected with it.

BOND GRAPHS

Bond graphs are based on the fact that the power or energy derivative of a dynamic system is composed of a number of terms. This number is equal to the number of interactions the system is capable of.

In full generality, the power P is a sum of products of intensities and extensity flows, the so called efforts and flows of BG. This sum characterizes the total power interactions in the system:

$$P = \sum_{i=1}^{n} e_i f_i \qquad (1)$$

where e_i stands for the efforts and f_i stands for the flows.

These considerations are related to the derivation of Tellegen's Theorem where all the junctions are placed within a conceptual boundary; they are valid in all domains of science [ATLAN 1973].

Thermodynamics, for example, takes into account the fact that matter can be converted into different forms by chemical reaction [THOMA 2006-1]. It describes the energy of a region of matter having n components with equations of the form:

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$$\dot{E} = T\dot{S} \sum p\dot{V} + \sum_{i=1}^{n} \mu_i \dot{m}_i \qquad (2)$$

Which can be visualized as in Fig. 1 :

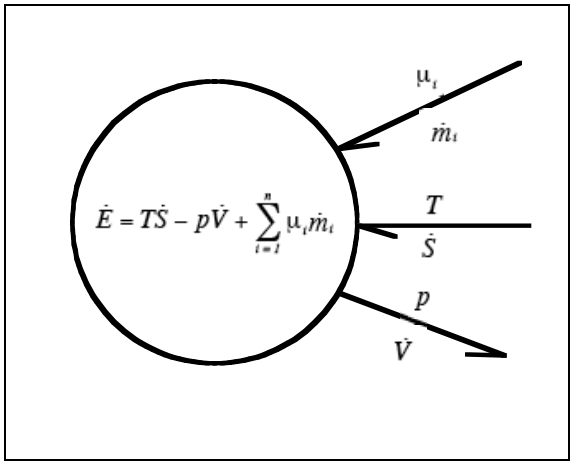

Fig 1 Equation 2 illustrated for a thermodynamic system. The power orientations correspond to the signs in equation 2.

This power equation shows the increase of energy in thermodynamic and chemical systems. It is an expression of the conservation of power, mainly chemical power.

The continuity equation for the ith extensity f_i within a control volume V under the effet of Sources and Sinks So_i, as well as Transfers Tr_i, takes the form:

$$\frac{-f_i}{-t} = So_i - Tr_i \qquad (2)$$

The transfer Tr_i of an extensity f_i is due to the product of the Conductance Co_i (the so called Siemens S of German speaking countries) times the difference of intensities or efforts e_i:

$$Tr_i = Co_i\left(e_i - e_{i0}\right) \qquad (3)$$

In full generality, the transfer of extensities is controlled by k intensities to take into account the cross effects among energy domains, the so called multiport R of BG:

$$Tr_i = \underset{k=1}{\overset{n-1}{—}} Co_{ik}\left(e_k - e_{k0}\right) \qquad (4)$$

Combining the continuity equation (2) with the transfer equation (4), we obtain the integral balance equation:

$$\frac{\Sigma f_i}{\Sigma t} = So_i \Sigma \sum_{k=1}^{n-1} Co_{ik}\left(e_k \Sigma e_{k0}\right) \qquad (5)$$

Being founded in the above transport theory, bond graphs allow the formulation of equations that describe any dynamic system. They state that the local value of the ith extensity is equal to its local density σ_i. By definition, the amount of a flowing extensitiy f_i contained in a volume equals the integral over the volume of the density σ_i of the extensity:

$$f_i = \int_V \int_i dv \qquad (6)$$

The total divergence So_i generated by all the sources and sinks within a volume can similarly be characterized in terms of densities σ_i:

$$So_i = \int_V \int_i dv \qquad (7)$$

The transfer Tr_i of extensive i through a surface F bounding a control volume V can be specified in terms of surface transfer densities $\mathbf{t}_i$:

$$Tr_i = \oint_F \boldsymbol{t}_i \, d\boldsymbol{f} \qquad (8)$$

If the intensities are scalar, the nabla operator supplies the gradient. The transfer density $\mathbf{t}_i$ of the ith extensity f_i is then stated as a sum of the products of the conductance Co_{ik} with the gradients of the individual intensities e_k:

$$\boldsymbol{t}_i = \sum_{k=1}^{n-1} Co_{ik}\ grad\ e_k \tag{9}$$

Introducing densities into the integral continuity equation (5) we obtain:

$$\frac{d}{dt}\int_V \sigma_i\, dv = \int_V \sigma_i\, dv - \oint_F \boldsymbol{t}_i\, d\boldsymbol{f} \tag{10}$$

This balance equation of the ith extensity refers to the entire control volume V.

The second term on the right hand side of equation (10) is a surface integral which can be converted into a volume integral. The surface integral of a vector over a closed surface F equals the volume integral of the vector's divergence over the volume enclosed by the surface (sometimes called the Stokes theorem):

$$\oint_F \boldsymbol{t}_i\, d\boldsymbol{f} = \int_V div\ \boldsymbol{t}_i\, dv \tag{11}$$

The variation of an extensity contained in a given volume is equal to the volume integral of the time variation of its density. We can add it to the volume integral of the volumetric transfer div$\mathbf{t}_i$ and substract the volume integral of sources and sinks densities.

Interchanging differentiation and integration with respect to time, we find the volume integral:

$$\int_V \left\{ \frac{\partial \sigma_i}{\partial t} + div\ \boldsymbol{t}_i - \sigma_i \right\} dv = 0 \tag{12}$$

For the integral to be identically zero irrespective of the limits of integration, the integrand itself must be equal to zero, which gives the general continuity equation upon which Paynter founded bond graphs [PAYNTER 1960], that is:

$$\frac{\partial \sigma_i}{\partial t} + div\ \boldsymbol{t}_i = \sigma_i \tag{13}$$

Replacing transfer densities $\mathbf{t}_i$ by the general transfer equation (9), we get the equation of any dynamic system, taking into consideration the relevant extensities and intensities of the reality under consideration and the source and sink densities due to the relation to the environment:

$$\frac{\partial \sigma_i}{dt} + div\left(\sum_{k=1}^{n-1} Co_{ik}\ grad\ e_k \right) = \sigma_i \tag{14}$$

If the considered reality is itself in motion, this motion carries along the extensities because they are properties of the bulk of reality: a global convective transfer must be added.

Convective transfer densities are expressed by the product of the extensity densities σ_i with the velocity of displacement $\mathbf{v}$, giving the convective transfer density $\sigma_i\mathbf{v}$ of the ith extensity:

$$\frac{\partial \sigma_i}{dt} + div\left(\sigma_i \mathbf{v} + \sum_{k=1}^{n-1} Co_{ik}\ grad\ e_k \right) = \sigma_i \tag{15}$$

This kind of mathematical relation is represented by bond graphs such as in Fig 2:

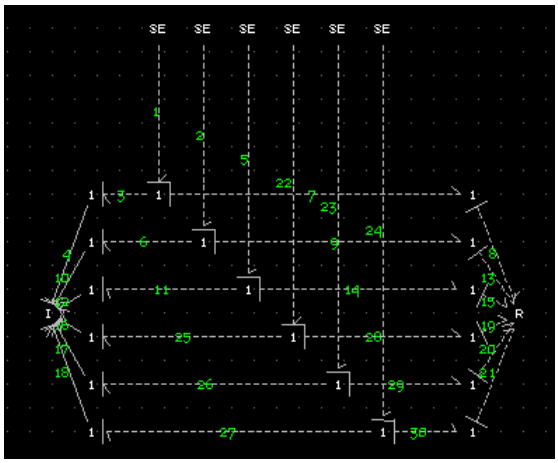

Fig 2 Representation of a dynamic reality with a bond graph (using the Camp-G software).

which give a complete knowledge of the dynamic reality at hand, including its dimensional analysis.

SIMILITUDE

Two dynamic realities are similar if the input vector x and the output vector y of both realities are related by the same function.

Coupled with dimensional analysis [SZIRTES 1997], which allows, for example, to understand why pressure can be conceived as well as force per surface or energy per volume, bond graphs give direct access to the concept of similitude between two dynamic realities [THOMA 2006-2].

For example, a thermal diffusion phenomenon is represented by the Fourier equation of heat conduction:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \qquad (16)$$

where T stands for temperature of the heat source and a stands for the temperature conduction coefficient.

And a matter diffusion phenomenon is described by the Fick equation of diffusion:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \qquad (17)$$

where c stands for concentration and D stands for the coefficient of diffusion.

Both phenomena are similar realities because they share the same mathematical model, a fact that can be used by people working in virtual reality.

This kind of comparison is very simple, with the state space equations automatically generated by bond graph software, as in Fig. 3:

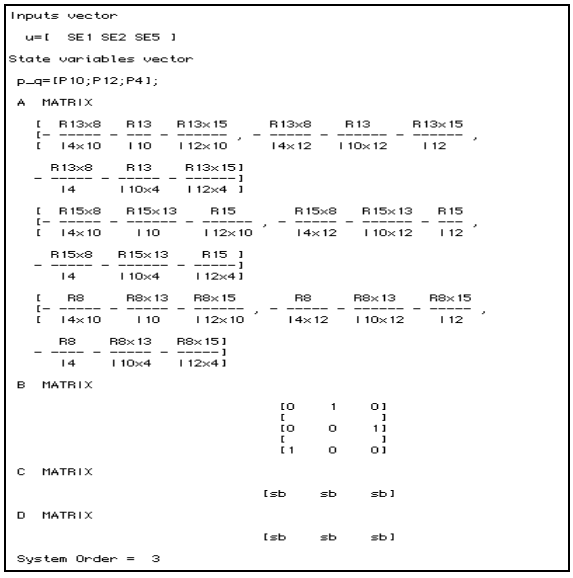

Fig 3 Example of a state space equation automatically generated by the Camp-G software.

Whatever the complexity of two considered realities, the necessary and sufficient condition of similitude between them is that the mathematical model of one be related by a bi-univoque relation to that of the other. In

other words that they have the same bond graph.

Modern computer programs allow the generation of simulators of the system from BGs, as in Fig.4 below. Moreover, programs like Systar (*http://www.systar.ch*) also allow the dimensional analysis of the variables describing the dynamic reality to be checked.

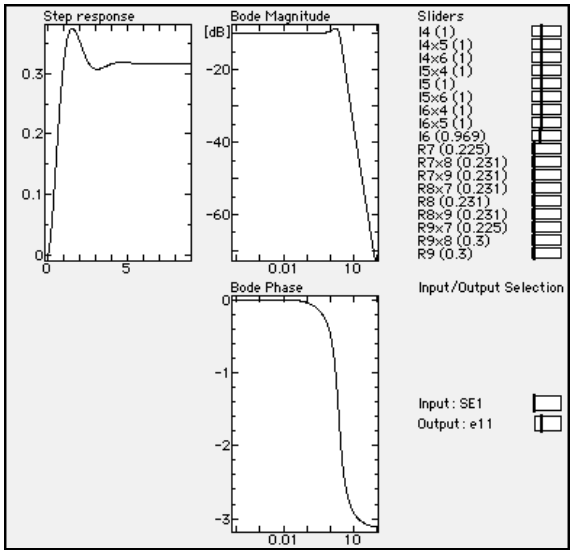

Fig 4 Front panel of an interactive simulator for a dynamic reality similar to the one in Fig. 2.

CONCLUSION

Bond graphs are graphical representations of dynamic realities. They consequently give a mathematical and computable formulation of the behaviour of the considered realities. They provide us with a tool that allows us to represent their dynamic conditions.

Moreover, this method of modelling and simulation can draw heavily on similitude between energy or power domains and experimentally established functional relationships among their invariants.

Finally, coupled with dimensional analysis, they allow criteria of similitude to be derived by comparison, providing solutions to many computer science virtual reality problems, within the domain of validity of the mathematical model.

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