A TWO DIMENSIONAL BOND GRAPH MODEL OF A SARCOMER

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ABSTRACT
This work concerns a Bond Graph modelling of the cardiac muscle cells named sarcomer. The dynamic behaviour of a set of connected sarcomers are analysed according to two directions. Some hypotheses are made to model the elements that cause the variations of a flat section taken in an auricle or a ventricle cavity of the heart. We assume that these dimensional variation result from the sarcomer dynamics. Bond Graphs models representing forces and motions of an isolated sarcomer are presented, followed by a Global one and its corresponding state space equation. To model this particular system, we take into account some of its electric, hydraulic, and mechanical multi field characteristic.

1. INTRODUCTION

Many models have been developed to represent skeletal or cardiac muscle. Some use mathematical formulation (Ayache 2002) or electromechanical schemes (Li 1996). Physiological and pharmacodynamic effects represent important fields of studies on the cardiac cell (Rocaries 2004). The Bond Graph formalism is usually used to model complex and mixed-domain dynamic systems (Karnopp1990) (Dauphin-Tanguy 2000). A simple Bond Graph model is proposed (Wojcik 2003) to study the dynamics of a skeletal muscle. Researches work currently on the whole cardiovascular system (Lefèvre 1999) (Zucarini 2003) using extensively this methodologie. Let us also mention (Cellier 2005) a work that contributes to the introduction of the Bond Graph methodology into medical domain modeling. Recently we have presented a Bond Graph model of a sarcomer (Fakri 05). We were precisely concerned with the mechanical contraction. The model, we proposed is shown figure1(c). A motor modeled as a gyrator port converts an electrical stimulus in rectilinear contraction and relaxation. It is used to simulate the dynamic behavior of the cardiac cells. To follow up this previous work, we propose an expansion of our previous unidimensional model by adding a second degree of freedom. Below a Sarcomer dynamic model is developed according to two main axes.
To end this introduction let us give a short description of what is usually assumed concerning the activation of the Sarcomer. It is composed by the adjustment of Myosin and Actin elements. Figure1 (a) shows the traditional scheme of the Sarcomer. The contraction of the Sarcomer occurs when the Actin and Myosin fibers creates a sliding linkages. Usually one considers that, as an actuator the Sarcomer contraction (Systole) is triggered by a potential change (depolarisation) of the cellular membrane, followed by an ordered movement of ions mainly sodium (Na^+), calcium (Ca^{++}), potassium (K^+), etc. The contraction is achieved by the stowing and the mutual slipping of the Actin and the Myosin filaments as shown in Figure 1(b).

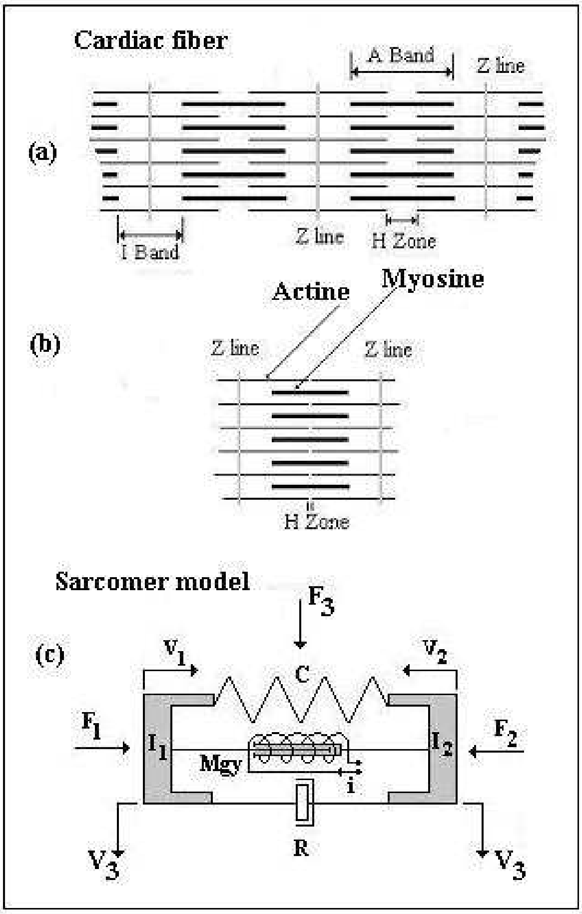

Figure1: Sarcomer grafic models

The shortening of the Sarcomer is characterized by a massive concentration of calcium ions on the tandem Actin-Myosin A mechanical tension takes place during those calcium ions movements. The relaxation phase of the cell (Diastole) is accompanied by an opposite flow of the calcium ions out of its membrane. The process ends with a repolarization of the membrane .A potential balance state results and is maintained until the next excitation.
The rest of this paper is organized as follows. Section 2 analyses the actions that implicate the sarcomer's

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movements. Section 3 presents the Bond Graph models and the state space equation of the global Bond Graph. Section 4 presents shortly a Bond Graph to Block Diagram transfer method. Some concluding remarks and directions for future work are given in the last Section.

2. THE SARCOMER AS PRINCIPAL ACTUATOR OF THE CARDIAC MUSCLE.

The sarcomer model, as an isolated rectilinear actuator is not satisfactory. Most studies agree to consider that the activity of these cells has as first consequence the dimensional variations of the muscular fibers. The second consequence is the surface and volume variations of the cardiac cavities. In the model that follows, we consider the sarcomer as a fiber component surronding a cavity section.

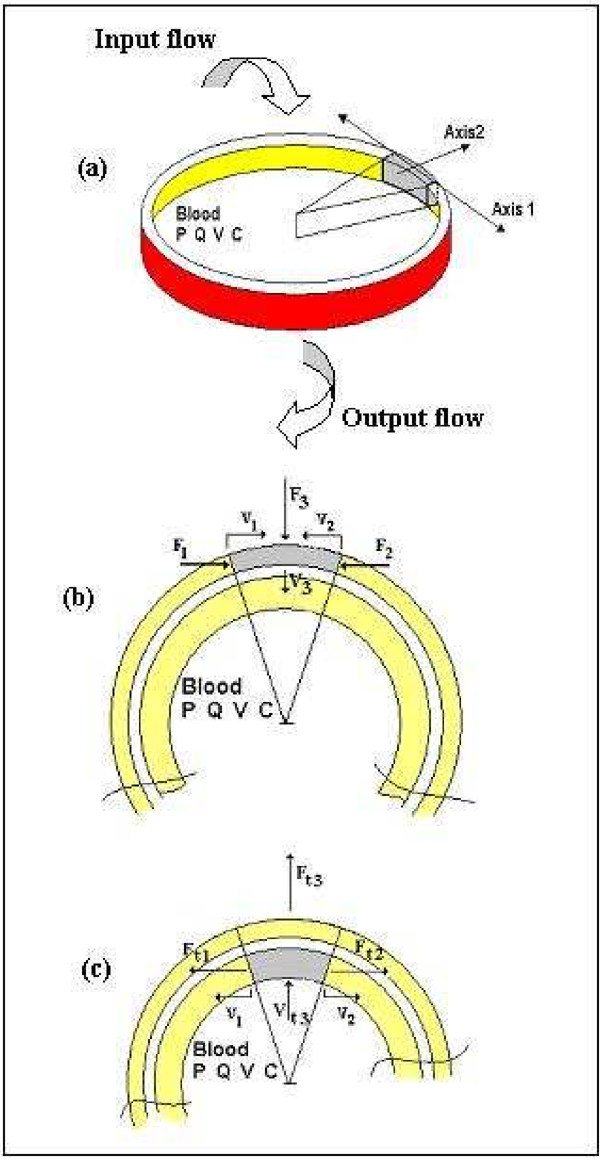

Figure 2: Forces and velocities along main axis

To establish the link between the activation of the sarcomers and the variations of cavity dimensions, we consider that the activation produce a composed movements. These movements follow two main directions anoted axis 1 and Axis 2 as indicated in the figure2 (a).

To simplify the analysis, we split up the cycle activities of the sarcomer in three phases:

The first phase is the bloof filling of the cardiac cavity, through an input flow valve. This phase corresponds to both of the sarcomer alternate relaxation and strain actions.

A second phase corresponds to the iso voluminal contraction. The cavity is full and tight during this stage of the sarcomer activation. A balance of strengths occurs during a short time preceding the following stage. The opening of the outflow valve starts the third phase. The blood ejection flow corresponds to the shortening and the displacement of the sarcomer. Cavity section and volume reduce during this last stage. Then the cycle restarts.

3. THE BOND GRAPH MODELS

From the analysis above we establish the Bond Graph representing the dynamics of the sarcomer during its activation cycle**.**

3.1 DYNAMICS DURING THE FILLING PHASE

At the time of the filling, we assume the existence of two kinds of forces. Normal forces of pressure are applied on the cavity wall. These forces are transmitted to the sarcomer along axis 2. Tangential constraints on the surface are also transmitted to the sarcomer along axis 1. These two efforts express strains according to the Laplace low related to membranes theory. The Bond Graph in figure 3 represents the dynamics of this filling phase.

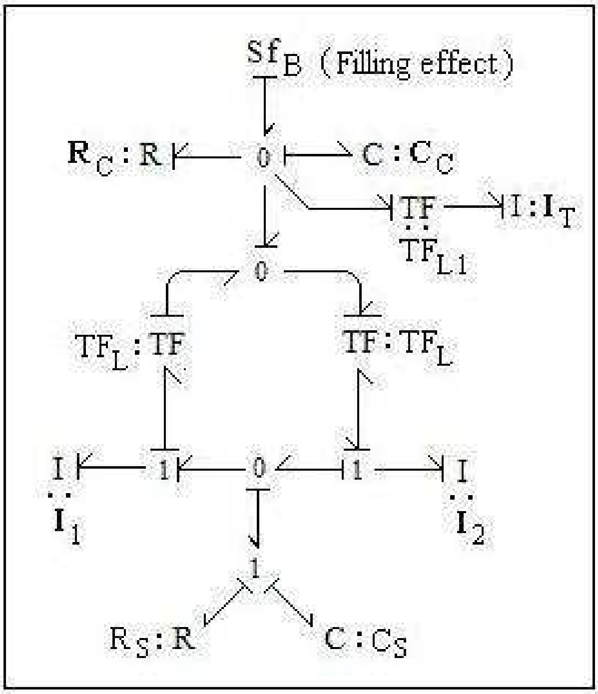

Figure 3: Sarcomer model during the filling phase

Essentially, the sarcomer cell expands along axis 1 by the spacing between inertias I_1 and I_2. The effort applied to the total inertia of the cell creates the movement along axis 2 directed to increase the cavity section.

3.2 DYNAMICS DURING THE CONTRACTION PHASE

This aspect of the dynamics of the sarcomer was developed in detail in a previous paper (Fakri 05). The Bond Graph model describes the electromecanical structure presented figure1 (c). The actuator is modelled by a gyrator port that converts the electric stimulus into two antagonistic forces. These forces are applied to the inertial elements of the cell. Capacitive (C) and Resistive (R) elements located between the two inertias ($I_1$and I_2) respectively dissipates or restores partialy the energy produced by the shortening of the cell.

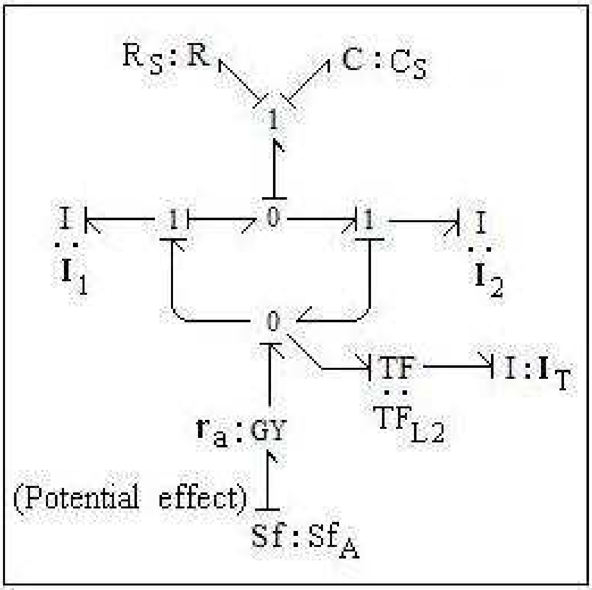

Figure 4: Sarcomer model during the contraction phase

The Bond Graph figure 4 represents the dynamics of the contraction. In this part of the cycle, the power variables (forces and velocity) are applied to the elements of the Sarcomer model along both of the two axis. Essentially the cell shortens along axis 1, reducing the gap between inertias I_1 and I_2. The effort applied to the total inertia I_T of the cell creates the motion along axis 2 directed to reduce the cavity section.

3.3 GLOBAL BOND GRAPH

To sum up, the motions of the inertial elements of the sarcomer model are dissociated. First the motions of the inertias I_1 and I_2 expands the muscular fiber along axis 1. The equivalent inertia I_T moves according to axis 2 and increases the section of the cavity. Second, in the opposite way the inertias I_1 and I_2 come closer and contract the muscular fiber along axis 1. The equivalent inertia I_T moves along axis 2 and reduces the cavity section. R_s and C_s represent rheological elements of the Sarcomer. R_c and C_c are the resistive en capacitive elements of the cavity volume acting on the sarcomer. All the transformers of the Bond Graph (TF_L, TF_{L1} TF_{L2}) convert hydraulic and mechanic efforts. These efforts are applied to the inertias of the Sarcomer along the two axes. The Global Bond Graph is shown figure 5. This model aggregates the previous models and illustrates all the analysed cycle.

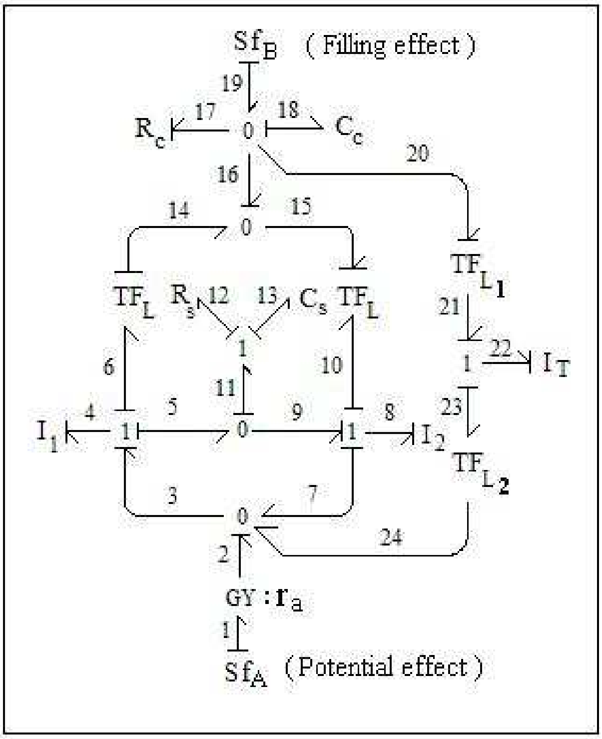

Figure 5: Global 2D Bond Graph model

3.4 STATE SPACE EQUATIONS

The state space equations of the global model may be directly extracted from its graphical representation.

$$\textbf{Equ. (a)} \qquad \dot{X} = AX + BU$$

$$\Downarrow$$

$$\begin{bmatrix} \dot{P}_4 \\ \dot{P}_8 \\ \dot{P}_{22} \\ \dot{q}_{13} \\ \dot{q}_{18} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{I_1} & \frac{R_s}{I_2} & 0 & -\frac{1}{C_c} & -\frac{TF_L}{C_c} \\ \frac{R_s}{I_1} & -\frac{R_s}{I_2} & 0 & \frac{1}{C_S} & \frac{TF_L}{C_c} \\ 0 & 0 & 0 & 0 & \frac{TF_{L1}}{C_c} \\ \frac{1}{I_1} & \frac{1}{I_2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{TF_{L1}}{I_T} & 0 & -\frac{R}{C_c} \end{bmatrix} \begin{bmatrix} P_4 \\ P_8 \\ P_{22} \\ q_{13} \\ q_{18} \end{bmatrix}$$

$$+ \begin{bmatrix} r_a & 0 \\ -r_a & 0 \\ -r_a.TF_{L2} & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} SF_A \\ SF_B \end{bmatrix}$$

The X-derivative contains the derivatives of the generalized energy variables related to the five ports that have an integral causality. The efforts applied to the inertias I_1, I_2 and I_T, and the velocity applied to the compliant elements C_s and C_c, compose the X-derivative vector. In the Bond Graph terminology, the momentums P_4 and P_8 relate to I_1 and I_2 respectively and the displacements q_{13} and q_{18} relates to the elements C_s and C_c. The state space equations modeling the dynamic behaviour of the sarcomer are given by Equ. **(a)**. The output equations representing the velocities V_i of the inertias (I_1, I_2 I_T) are expressed in Equ**. (b)**.

$$\textit{Equ. } (b) \qquad Y = CX$$

$$\Downarrow$$

$$\begin{bmatrix} f_4 \\ f_8 \\ f_{22} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{I_1} & \frac{1}{I_2} & \frac{1}{I_T} & 0 & 0 \end{bmatrix} \begin{bmatrix} P_4 \\ P_8 \\ P_{22} \\ q_{13} \\ q_{18} \end{bmatrix}$$

Simulations of this system can be made by tools that use the graphical Bond Graph model directly (Granda 2002). State Equations programming can also be exploited. Conversion into block diagram is also used, so we often use a method that facilitates and accelerates this conversion.

4. TRANSFER FROM BOND GRAPH TO BLOCK DIAGRAM

A particular method, we suggested (Fakri 97), make the transfer easier and quiker. It allows building the Block diagram model from a direct reading of the Bond Graph, without writing beforehand all equations of junctions. One can thus benefit from all the potentiality of a numerical computational tool supplied with a Block diagram graphical interface. In this short reminder below, we assume that the reader is familiar with the Bond Graph elements. The achievement of the transfer to block diagram uses the following four notions we have introduced: The **Owner Bond** of junction, the **Owner Block** of junction, the **internal** and **external** bonds.

Therefore we identify for each junction a part or all the elements defined below.

The owner bond of junction: It is the unique bond that imposes by its causality, the common power variable on the considered junction.

The internal bonds of junction: This is R, I or C element bond that requires a constitutive law. They also have the common power variable as input. They are always supplied with a local feedback loop.

The external bonds: These are sources and bonds exchanging power with other junctions directly or through transformer (TF) or gyrator (GY).

The owner block of junction: It is a summing block diagram. It achieves the power conservation. As ouput, it always has the power variable (effort or flow) of the owner bond. As input, it has the power variables of the internal and external bonds of the considered Bond Graph junction. This block makes the algebraic sum of these input signals.

The constitutive blocks: These are the blocks representing the mathematical relations (constitutive laws) associated witk the power variables applied to the Bond Graph elements that composes the junction.

The figure 6 illustrates the elements defined above.

Each junction corresponds to an owner block and has the same number. The block diagram model is achieved by a direct reading of the Bond Graph. Bonds and the junctions are numbered as a preliminary. More details can be found in the paper dedicated to this method (Fakri 97).

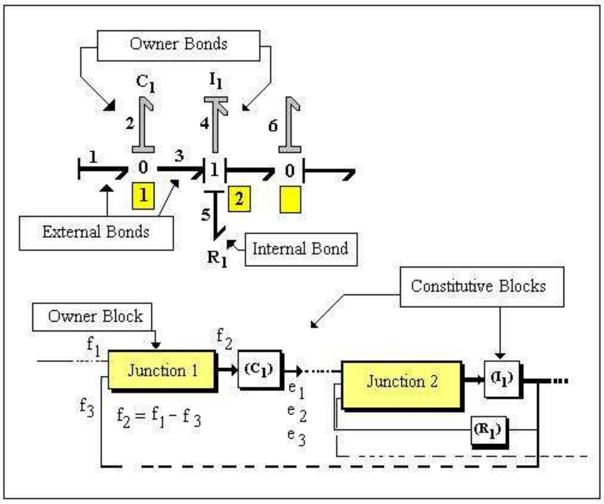

Figure 6: Example of Transfer elements

5. CONCLUSIONS AND FUTURE WORK

A 2D model of the sarcomer cell is proposed in this paper. The Bond Graph methodology is used as a powerful alternative to model the dynamics of the elementary cardiac cell. The model is built based on the analysis of the blood flow through the cardiac muscle and the electric stimulus on the cells. The three phases cycle of the cardiac activity are linked to the sarcomer mobility along two axis. Further developments beyond this stage of the modeling work can be continued in two main directions:

To collect realistic simulation values.

To model an automat needed to control the working phases of the cardiac cycle.

This automat must be implemented and linked to the Bond Graph to simulate and improve the 2D model proposed and then enable the study of a 3D model of the whole cavity.

REFERENCES

Ayache N, Chapelle D, F. Clément, Y Coudière, H. Delingette, J.A. Désidéri, M. Sermesant, M. Sorine, and J. Urquiza 2002, “Towards Model-based Estimation of the Cardiac Electro- Mechanical Activity from ECG Signals and Ultrasound Imags”, *ESAIM*: Proc., Vol. 7, pp 1-10.

Cellier F.E. and Nebot A. , 2005, Object-oriented Modeling in the service of Medicine. A: *Proceedings of Asia Simulation Conference / the 6th International Conference on System Simulation and Scientific Computing*. International Academic Publishers, pp. 33-40.

Dauphin-Tanguy G. & col. 2000, "*Les Bond Graphs*", Hermes édition.

Diaz-Zuccarini V. 2003 "*Etude des Conditions d'efficacité du ventricule gauche par optimisation téléonomique d'un modèle de son fonctionnement*". Thèse Doctorat, Ecole Centrale de Lille – Univ des Sciences et Technologie de Lille2.

Fakri A., Rocaries F . and Carriere A. 1997 "A simple method for the conversion of bond graph models in representation by block diagrams". *International Conference on Bond Graph Modelling and Simulation. ICBGM'97*. Phoenix, Arizona, 15-19.

Fakri A. and F. Rocaries F. 2005. "Study of the cardiac muscle dynamics utilising Bond Graph methodologie". *International Conference on Bond Graph Modelling and Simulation. ICBGM'05*. New Orleans, Louisana, 281-284.

Granda Jose J. 2002 "The role of bond graph modeling and simulation in mechatronics systems: An integrated software tool: CAMP-G, MATLAB–SIMULINK, *Mechatronics*, Vol.12, 1271-1295.

Karnopp D.C., D.L. Margolis and R.C. Rosenberg 1990 "*System dynamics: A Unified Approach*" 2nd édition, Wiley interscience Publication.

Lefèvre J, L. Lefèvre, Couteiro B. 1999 "A Bond Graph model of chemo-mechanical transduction in the mammalian left ventricular. *Simulation Practice and theory 7, ELSEVIER*, pp 531-552

Li J.K-J , J.Jung Wang, G. Drzewiecki, 1996. "Computer Modeling of Non-Adjacent Regional Ischemic Zones on Ventricular Function". *Comput. Biol. Med*. Vol. 26, No 5, pp 371-383.

Rocaries F, Y.Hamam, R.Roche, M.Delgado, R.Lamanna, F.Pecker, C.Pavoine and H.Lorino 2004, "Calcium Dynamics in Cardiac myocites:A Model for Drugs Effect Description". *Simulation modelling and theory Vol.12 issue2, Elsevier Sc*. pp.93-104.

Wojcik L.A. 2003 "Modeling of musculoskeletal structure and function using a modular Bond graph approch" *Journal of the franklin Institute* 340,pp 63-76.

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