# AUTOMATED RECONSTRUCTION OF BOND GRAPH MODELS BASED ON FREQUENTIAL SPECIFICATIONS

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bond graph, reconstruction, junction structure

**ABSTRACT**

This study proposes a method for automatic reconstruction of models that present modelling inconsistencies.
It first describes a general procedure, derived from the bond graph modelling process, which builds generic (independent of physical and technological domains) models. By applying different elimination criteria we extract those models that comply with our demands. This general algorithm is adapted for the reconstruction of a tympanometer's model with a frequential problem observed after the construction phase.

## INTRODUCTION

Nowadays the systems to be modelled have become increasingly complex, and need accurate models in order to understand and control their behaviour.
As engineering systems today are multi-domain, they require the combined work of several engineering disciplines. In order to automate the design or revision of multi-domain systems, the bond graph methodology has been chosen.
The main concern of an engineer confronted with a modelling problem is to make an abstraction of the system he needs to model. It is a difficult task to determine which aspects or properties are relevant for the system at hand. Sometimes these choices may be the source of models with inconsistencies that will fail in the analysis or control phase. Several physical aspects of the system (friction, fluid inertia, etc) that were not taken into account may be the cause of an inconsistent model.
Other problems may appear after the modelling phase when certain aspects of the system do not comply with the specifications. This proves that modelling is an iterative task and the model proposed is constantly adapted and revised until it satisfies the designer's needs and that a large part of the modelling process consists in revision and adaptation.
This study proposes an automation of this aspect of modelling.
Following the well-known process of bond graph modelling, we want to provide models that respond to specifications, expressed here in terms of frequency and transfer function.
Section 2 shows the first step of the procedure, the structure generation. All possible junction structures with a certain number of 0- and 1- junctions are provided (Pirvu et al. 2005). In order to avoid duplicate structures, several algorithms are thus applied. By using rules of equivalence (Lamb et al. 1993), redundant structures are eliminated.
Section 3 deals with causality rules and element placement.
In section 4 the specifications are expressed in terms of frequency response. **All** different structures verifying the previously defined rules and satisfying the specifications are obtained. The result is thus a set of complete bond graph models, which can be, in a next step, interpreted in terms of technological devices in different physical domains.

## STRUCTURE GENERATION

### Building of the structures

The first task is to develop all possible junction structures with a certain number of 0- and 1-junctions. These junction structures are like skeletons as they consist only of junctions and their connecting bonds (Ort and Martens 1973; Birkett and Roe 1989, 1990).
The bond graph designing process has been developed around a set of basic rules described as assumptions $A_1$ and $A_2$:
**$A_1$:** No two consecutive junctions of the same type are allowed
**$A_2$:** All external elements are added on the 1- junctions (derived from the systematic procedure of constructing bond graph models described in (Karnopp and Rosenberg 1975))
For representing a skeleton, a Boolean matrix is introduced where a connection between a 0-junction and 1-junctions is signalled by adding the value "1" in the corresponding cell of the matrix as shown in figure 1.

$$0_1 \;\text{———}\; 1_1 \;\text{———}\; 0_2 \;\text{———}\; 1_2 \qquad M = \begin{array}{c c} & \begin{array}{cc} 1_1 & 1_2 \end{array} \\ \begin{array}{c} 0_1 \\ 0_2 \end{array} & \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \end{array}$$

Figure 1: A BG Skeleton and its Structure Matrix

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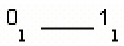
The algorithm starts the generation process from a minimal structure. This minimal structure may be the matrix $M$ = (1) which represents the most simple junction structure with only two junctions as shown in figure 2(a) or it may be a more complex one if the modelling process is not at the beginning.
As we are discussing a tool that adapts and reconstructs inconsistent models, our starting point will be a more complex junction structure that corresponds to an already existing model like in figure 2(b).


(a) Basic structure

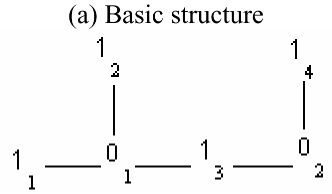
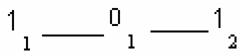

(b) A more elaborate structure

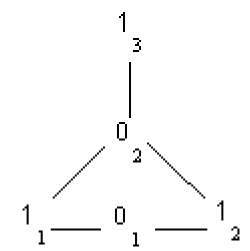
Figure 2**:** Starting BG Structure

It is preferable to begin this process with an already existing skeleton that represents the most vital or simple to design part of the model. In every modelling case, the engineer can always determine a certain part of the model considered the "backbone" of the whole design.
Every already developed structure is considered as a starting point for further structures with an increasing number of junctions.
Assumption $A_2$ states that elements (R, I, C, Se, Sf, De, Df) cannot be connected directly to 0-junctions.
To speed up the generation process and to permit element placement, we will add a 0-junction coupled with a 1-junction.
From figure 3 (a), adding a 0-junction leads to figure 3 (b) with a new matrix M.



$$M = \begin{array}{c} \\ 0_1 \end{array} \begin{array}{c} \begin{array}{cc} 1_1 & 1_2 \end{array} \\ \begin{pmatrix} 1 & 1 \end{pmatrix} \end{array}$$

(a) Initial Structure and Corresponding Matrix



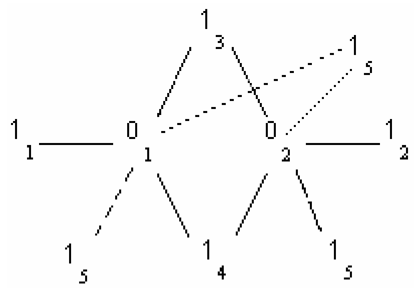
$$M = \begin{array}{c|ccc} & 1_1 & 1_2 & 1_3 \\ \hline 0_1 & 1 & 1 & 0 \\ 0_2 & 1 & 1 & 1 \end{array}$$

(b) Final structure and corresponding matrix

Figure 3: Adding a New 0-junction

As it can be observed, adding junctions means adding rows and columns to already existing matrices (rows for 0-junctions and columns for 1-junctions).

A junction added to an already existing structure can be connected in multiple ways as in figure 4. Every time we obtain a new valid structure. By exploring all possibilities of connecting a new junction, the algorithm will provide all new structures derived from a given base.


(a) Adding a 1-junction

$$M_1 = \begin{array}{c|ccccc} & 1_1 & 1_2 & 1_3 & 1_4 & 1_5 \\ \hline 0_1 & 1 & 0 & 1 & 1 & 1 \\ 0_2 & 0 & 1 & 1 & 1 & 0 \end{array} \quad M_2 = \begin{array}{c|ccccc} & 1_1 & 1_2 & 1_3 & 1_4 & 1_5 \\ \hline 0_1 & 1 & 0 & 1 & 1 & 0 \\ 0_2 & 0 & 1 & 1 & 1 & 1 \end{array}$$

$$M_3 = \begin{array}{c|ccccc} & 1_1 & 1_2 & 1_3 & 1_4 & 1_5 \\ \hline 0_1 & 1 & 0 & 1 & 1 & 1 \\ 0_2 & 0 & 1 & 1 & 1 & 1 \end{array}$$

(b) Corresponding matrices

Figure 4**:** Developing Structures

From the bond graph point of view, structures $M_1$ and $M_2$ are equivalent even if their matrices are different. As the process of structure developing provides all possible configurations, the problem is how to detect and eliminate these redundant structures.

**Eliminating redundancies**

Each junction has associated a number of internal bonds that connect it to other junctions. Each time a matrix is developed, it has to be compared with previous ones to decide whether it is redundant or not. We do this by comparing the number of internal bonds for junctions of the same type. At the matrix level this information consists in summing each row and each column. All sums per rows (columns) are retained in a vector.
For the matrices described below the numbers are:

$M = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ (2, 1) for 0-junctions; (2, 1) for 1-junctions

$M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (1, 2) for 0-junctions; (2, 1) for 1-junctions

We proposed the following procedure:

**Procedure $P_1$**

-Step 1

- If vectors for the same type of junctions are different (order is not important as a change in order means a permutation of junctions), the second structure is original and is kept as a possible configuration. Otherwise it is not possible to conclude, and thus go to step 2.

-Step 2

- Calculate all the minors for the considered matrices to be compared.

Example:

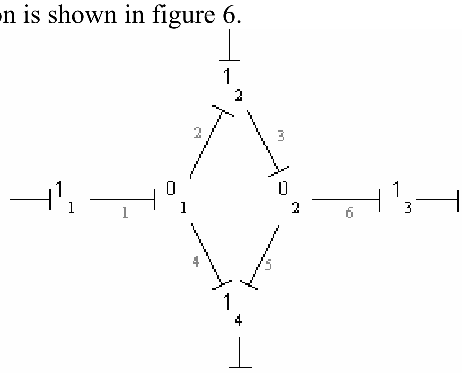
$$M = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ and its minors (absolute value)}$$

$$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1;\ \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1;\ \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$$

If they are the same, the structures are equivalent and only one of them has to be kept.

## CAUSALITY AND ELEMENT PLACEMENT

The generation procedure has provided all possible and unique configurations of 0 and 1-junctions.
In the process of transforming these structures into bond graphs, the next step is about applying causality.
Causality is represented by introducing in the M- matrix (+) and (-) signs: a causal stroke "far" from a 1-junction is coded with –1 and a causal stroke "close" to the 1-junction is coded with 1. The rule for 1-junctions states that one causal stroke "far" will impose all the others "close". This means that only one (–1) is permitted for every column. For rows the logic is reversed: one (–1) means a causal stroke "close" to the 0-junction and this imposes all other causal strokes "far" i.e. coded with (+1).
Every 1-junction has several internal bonds (connecting them to 0-junctions) and one or more external bonds for supporting elements. The external bonds are treated as 0-junctions and are represented by new rows in this matrix, coded as described for 0-junctions.
An example of this convention is shown in figure 5. The last row of the matrix corresponds to the external bond of each 1-junction.
When there is no predefined model, the algorithm generates all causality solutions i.e. constructs all matrices of causality for every structure. Notice that for the structure in figure 5 another possible causality solution is shown in figure 6.

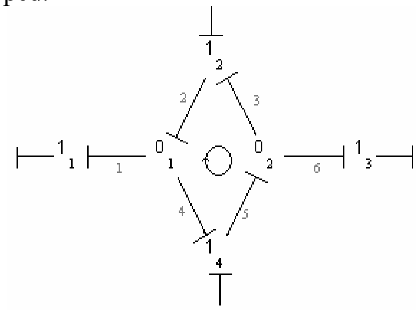


$$\begin{array}{c} \\ 0_1 \\ 0_2 \\ ext_1 \end{array} \begin{array}{c} \begin{array}{cccc} 1_1 & 1_2 & 1_3 & 1_4 \end{array} \\ \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \end{array}$$

$$\{S_e, C, D_f, R, I_{der}\} \quad \{S_f, I, D_e, R, C_{der}\}$$

Figure 5: Example of Causality Representation

Another assumption was introduced at this point of the study:
**A$_3$**: A causality loop is not allowed (an infinite sum may be obtained when calculating the gain of the loop)
Because of assumption A$_3$, a solution as shown in figure 6 is dropped.



$$\begin{array}{c} \\ 0_1 \\ 0_2 \\ ext_1 \end{array} \begin{array}{c} \begin{array}{cccc} 1_1 & 1_2 & 1_3 & 1_4 \end{array} \\ \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \end{array}$$

$$\{S_f, I, D_e, R, C_{der}\} \quad \{S_e, C, D_f, R, I_{der}\}$$

Figure 6: One causality Assignment to be rejected

For each causality solution, there are specific elements that fit in. A (-1) in the causality matrix indicates that a member of the "outgoing flow" family $\{S_f, I, D_e, R, C_{der}\}$ will be chosen as element for that 1-junction, (+1) indicating a candidate from the "outgoing effort" family $\{S_e, C, D_f, R, I_{der}\}$. For each causality solution the algorithm provides all possible configurations of elements. This idea is followed also in the case of a partially developed model.
Models are reconstructed following two procedures:

- adding new elements
- adding new junctions

These two steps are intercalated in the case of a total reconstruction of a bond graph model.
In the process of adding new elements each 1-junction is considered a support for new storage elements.
If the 1- junction already detains external elements that were considered vital, the algorithm will not permit the addition of elements of the same type. In the example of figure 7 only an I-element in derivative causality may be introduced.

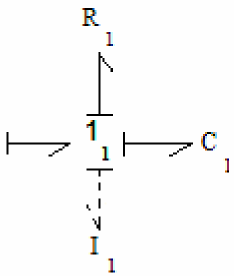



Figure 7: Adding Dynamic Elements

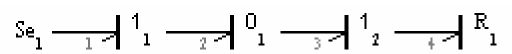
If the procedure of adding new elements has not given a solution to our specifications, a modification in structure is triggered. The algorithm is reloaded from the junction structure phase, where an already existing structure is completed with new junctions. The introduction of TF-junctions is also permitted at this stage if the algorithm considers a new physical domain should be explored.

## TRANSFER FUNCTION

We have chosen to introduce at this stage the power transfer direction and the corresponding half arrows. As we discuss here about SISO models, we set the following assumption:

**$A_4$:** the power propagates from the source to every branch of the structure.

For the power flow representation, the same logic as for causality is used, by introducing a power flow dedicated matrix whose terms are equal to (+1) if the half-arrow enters the 1-junction, (-1) if the half-arrow leaves the 1-junction. The half-arrow for external bonds always points out of the sources and into the other elements.




$$A = \begin{array}{c c} & \begin{array}{cc} 1_1 & 1_2 \end{array} \\ \begin{array}{c} 0_1 \\ Ext \end{array} & \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \\ & \begin{array}{cc} \downarrow & \downarrow \\ S_e & R_1 \end{array} \end{array}$$

Figure 8: a) Power Oriented BG and its Corresponding Matrix

Using causal loops and path gains, the Mason's rule may be implemented. For each generated solution we calculate the transfer function and compare it with the one given as specification.

## APPLICATION

What is described above is a general procedure for the automatic design of bond graph models that exhibit a desired dynamic behaviour. This procedure is adapted for the reconstruction of a device that determines the health of the internal ear. This example was presented by D. Margolis in (Margolis 2002) as a design error that could have been avoided if further analysis had been made during the conception phase. A solution was found after the production phase and we want to prove that this solution could have been automatically indicated by the proposed algorithm.

The device called tympanometer uses an acoustic signal to detect the health of the internal ear. Its schematic description is shown in figure 10.

A small rubber cone is introduced into the ear and a motor-pump system pressurizes the ear cavity to 20 mm of Hg. Then the pressure is backed down with 5 mm of Hg decrements and an acoustic signal of 226 Hz is transmitted into the ear cavity. The acoustic pressure is measured by a microphone and analyzed to determine possible anomalies.

The bond graph model of the right part of the system (in the box) is described in figure 9. The descriptions of compliances, fluid inertia and fluid resistance are as following:

$$I_f = \frac{\rho L}{A_t}\ ,\ L = tube\ length;\ A_t = surface\ of\ the\ tube$$

$$C_i = \frac{V_i}{\rho v_s^2},\ V_i = chamber\ volume;\ \rho = air\ density;$$

$$v_s = speed\ of\ sound\ in\ the\ air\left(343\ {}^{m}\!/_{s}\right)$$

$$R_f = \frac{8\eta L}{\pi r^4},\ L = tube\ length;\ r = radius\ of\ the\ tube,$$

$$\eta\text{=}dynamic\ viscosity\ of\ air\left(18\cdot 10^{-6}\ Pa\cdot s\right)$$

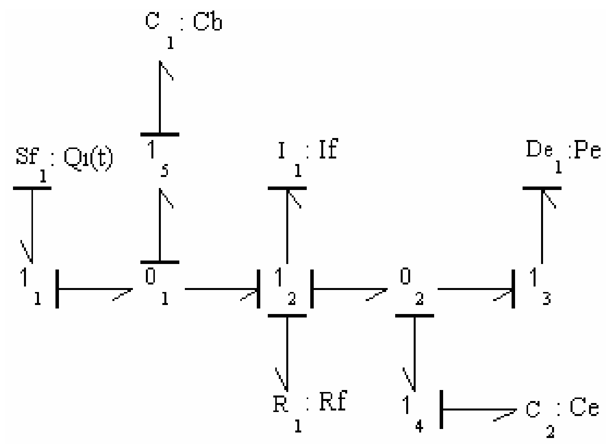



Figure 9: Bond graph Model of Tympanometer

In the production phase this device was not functioning as supposed and needed a more elaborate analysis.

Apparently a "slip-stick" friction of the piston-cylinder was injecting noise into the ballast volume.

The transfer function of the dynamical system in figure 9 was calculated as described below:

$$H_1(s) = \frac{P_e}{Q_1}(s) = \frac{{}^{1}\!/_{C_e C_b I_f}}{s\left(s^2 + \frac{R_f}{I_f}s + \frac{1}{I_f}\left(\frac{1}{C_e} + \frac{1}{C_b}\right)\right)}$$

where $P_e$ is the pressure in the ear given by $D_{e1}$

According to actual values provided for this device the natural frequency was found to be 220 Hz.

$$\omega_n^2 = \frac{1}{I_f}\left(\frac{1}{C_b} + \frac{1}{C_e}\right) \approx 440\pi \text{ close to 226 Hz}$$

The system's dynamics were amplifying noise at almost the exact frequency necessary for the treatment of the signal.

The solution to this problem may be the introduction of a pair of zeros in the transfer function that would eliminate the desired frequency.

$$H_2(s) = \frac{P_e}{Q_1}(s) = \frac{k\left(s^2 + \omega_0^2\right)}{s\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)}$$

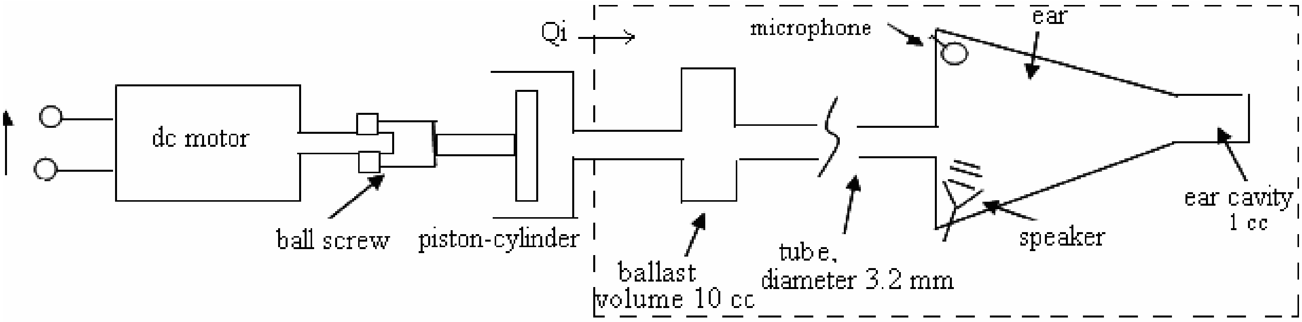
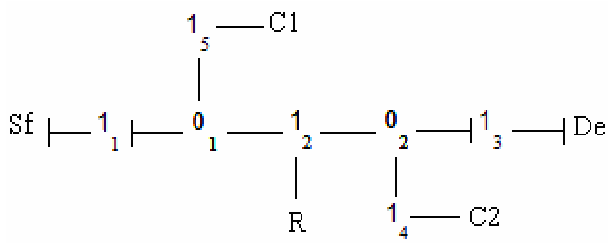



Figure 10: Schema of the Tympanometer

The actual value of this zero has to be adjusted to eliminate the frequency of 226 Hz and avoid the propagation of noise at this frequency.
The bond graph model from figure 9 can be used as a starting point for further models that should provide a double zero to this system. The desired transfer function is $H_2(s)$ and considered as a specification for the generation procedure.
Information must be provided to the generation procedure in order to establish:

- Which part of the initial model should be reconstructed and implicitly which part is considered vital and should not be changed?
- How radical this reconstruction should be, meaning only an addition of storage elements or also a junction structure modification?
- What are the specifications for the new dynamic behaviour?

Because this device was already in the production phase when the malfunctioning was observed, the main concern is to keep as much as possible from the original system. Our goal is to obtain a model that introduces the zeros needed, preserves the order of the model and triggers minimal modifications in the design of the device. Since no modification in structure is needed yet, the algorithm will not take into consideration the stage of developing a new skeleton.
If modification in structure should be permitted, the algorithm would load the minimal structure, figure 11, and start developing new possible ones (adding junctions).
The device itself introduces some constraints that are considered as vital parts: we inject air (Sf) into the ballast chamber (C1); we measure the pressure (De) into the ear cavity (C2) and there is a tube introduced in the ear cavity (R). All these components have already been produced and therefore the designer prefers them unchanged.
Since we already have a source, the power flow matrix is imposed. The source of air will also impose it's causality on the first 1-junction and the third 1-junction has its causality imposed by the second 0-junction. The rest of the causality matrix will be completed automatically, as well as new storage elements.

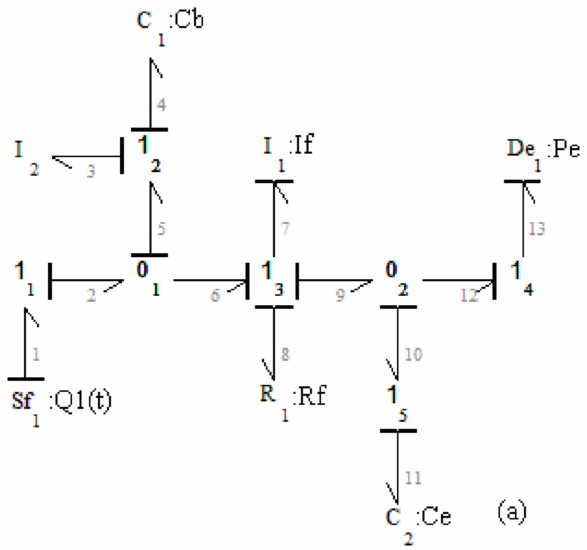



$$M = \begin{array}{c} \\ 0_1 \\ 0_2 \\ \text{Ext} \end{array}\begin{array}{c} \begin{array}{ccccc} 1_1 & 1_2 & 1_3 & 1_4 & 1_5 \end{array} \\ \begin{pmatrix} 1 & \pm 1 & 0 & 0 & \pm 1 \\ 0 & \pm 1 & 1 & \pm 1 & 0 \\ -1 & \pm 1 & -1 & \pm 1 & \pm 1 \end{pmatrix} \end{array}$$

Figure 11: Minimal Junction Structure and Imposed Causality

As it can be observed, the design of the tympanometer imposed certain constraints on the generation procedure, each step of the algorithm working with some predefined information.

The generation procedure provided only two possible solutions that correspond to specifications:



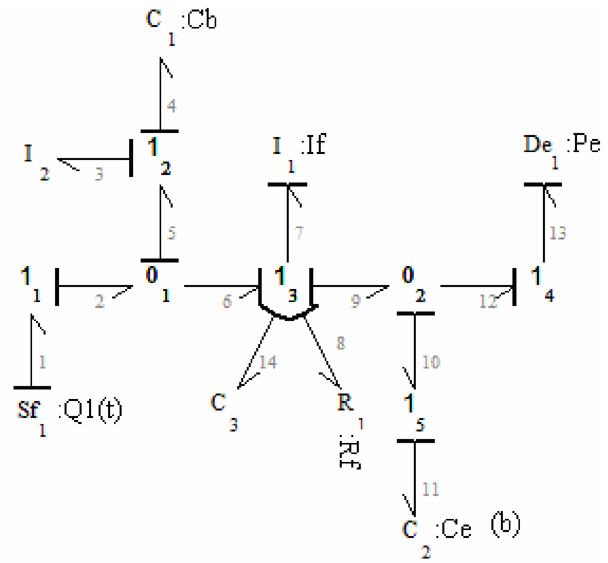



Figure 12: Possible Solutions

Model 12(a) introduced an additional I-element ($I_2$) that provided a pair of complex zeros necessary for the reject of a certain frequency. Indeed the new transfer function is:

$$\frac{P_e}{Q_i}(s)=\frac{\frac{1}{C_e}\frac{1}{(1+{}^{I_f}\!/_{I_b})}\left(s^2+\frac{1}{I_bC_b}\right)}{s\left(s^2+\frac{{}^{R_f}\!/_{I_b}}{1+{}^{I_f}\!/_{I_b}}s+\frac{1}{(1+{}^{I_f}\!/_{I_b})}\left(\frac{1}{I_bC_b}+\frac{1}{I_bC_e}\right)\right)}$$

with $\omega_0^2=\dfrac{1}{I_bC_b}$, $I_b$ corresponding to $I_2$

The volume of the ballast has been preserved but is separated from the original tube by a smaller tube the length of which we may calculate so that to reject 226Hz. It is precisely the solution found by the designer for the redesign of the device and it corresponds to a Helmholtz oscillator. The side branch was easily installed without too much affecting the schematic of the device as seen in figure 13:

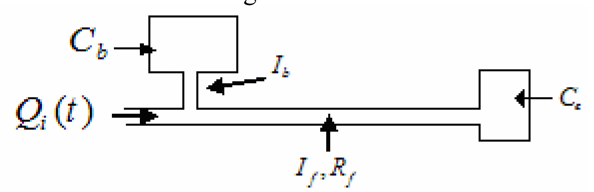



Figure 13: Redesign Solution

Solution 12(b) introduces the I-element for the pair of zeros and also a C-element (C3). This solution has four dynamic elements. An ulterior analysis of this system proved that a simplification null pole-null zero is made and that the C3 is not controllable. Due to this stability and controllability issues the second solution is not considered viable.

## CONCLUSIONS

In [Margolis2002] the applicability of the bond graph modelling methodology during the conception phase of a design process is proved. The case of a modelling error in a hand held medical instrument has been treated and bond graph modelling has proved to be an excellent tool for extracting state variables, equation formulation and simulation.
We want to point out another capability of this method, that of a reconstruction tool, which may indicate to less experienced modellers, a possible solution in case of an inconsistent with specifications design. In the case of the medical instrument the most simple and efficient solution was found by the algorithm and it corresponds exactly to the solution pointed out by the engineers.

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