AN M^X/G/1 RETRIAL QUEUE WITH UNRELIABLE SERVER AND VACATIONS

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Abstract: In this paper**, w**e consider retrial queueing systems with batch arrivals in which the server is subject to controllable interruptions (called vacations) and random interruptions (breakdowns). No specific assumption is taken regarding probability distributions of parametric random variables. The purpose of this work is to show effect of above mentioned parameters (in particular retrial and breakdown parameters) upon main performance measures of interest. Next, we study some optimal control problems of vacation and retrial policies.

Keywords- Retrial queues, vacations, batch arrivals, breakdowns, cost model, optimal control, reliability.

1.INTRODUCTION:

Queueing (or service) systems arise in modelling of many practical applications related to Computer Sciences: Communication, Production, Human-Computer Interactions, and so on. In this paper we consider queueing systems which take into consideration additional phenomena:

(i) Batch arrivals of customers: in many computer systems, the message is transmitted by packets (frames).

(ii) Repeated attempts of unsatisfied customers: see the bibliographical paper of Artalejo 1997

(iii) Idle time use of servicing device through introducing periods (for example maintenance actions in order to prevent the risk of failure): see the survey of Doshi (1986)

(iv) Random interruptions due server breakdowns or other priority tasks.

Similar models have been used in concrete applications as the modelling of Digital Cellular Mobile Networks [Sun Jong Kwon, 2001], Local Area Networks with star topology [Janssens,1997]and so on. However all the models used there, neglected breakdown process.

In this work we also study the optimal control of vacation and retrial policies. Some attempts have been made in this direction by Artalejo (1997) in the case of Poisson arrival process and by Aissani (2000) in the case of batch arrivals. However, in both papers they considered the case of constant retrial policy with an absolutely reliable server.

Next section is meant for description of the model. Section 3 concerns the analysis part of the problem where we obtain probability distribution of the system state. These results are obtained by the method of supplementary variables which is well known in Queueing Theory. So, we will provide only the necessary elements. This study confirms some decomposition properties showing the effect of vacations and retrials. Finally, we consider the problem of the optimal control of vacation policy (section 4) and retrial policy (section 5). We conclude the study by some numerical examples.

2. MODEL DESCRIPTION:

We consider a single server queue where batches of customers arrive according to a compound Poisson process. If an incoming batch finds the server idle, one of the batch members immediately begins service and the rest of customers in that batch join the retrial group (a sort of queue with infinite capacity also called: the orbit) and seek for service individually after a random amount of time. The server is subject to random breakdowns with rate θ. Whenever the server fails, it is immediately repaired. If an incoming batch finds the server unavailable (i.e. busy by the service of a certain customer, out of order or in vacation), then all customers in the batch join the orbit. Any customer accepted for service upon arrival or on retrial leaves the system forever after service completion. We consider the following policy to access the server from orbit. If the orbit is not idle at some instant, then a random customer is chosen to occupy the server after a random amount of time Λ. We assume that the server takes a random vacation

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each time the system is empty. If the server returns from vacation to find one or more customers waiting in orbit, he works until the system empties, then begins another vacation. If the server returns from a vacation to find no customers in orbit, he begins another vacation immediately. We assume that all the considered variables are mutually independent.

Acronyms

PDF Probability Distribution Function
PGF Probability Generating Function
LST Laplace-Stieltjes Transform
NBUE New Better than Used in Expectation

Notation

λ=the batch arrival rate
X=size of an arrival batch
G(z)=PGF of X
$g_k=E(X^k)$=kth order moment of X
S=service time random variable
Λ=Retrial time
V=vacation time random variable
θ= rate of breakdowns
W=repair time of a breakdown
H(x), R(x), V(x), W(x) The PDF of the random variables S,Λ,V, W.
h(s), r(s), v(s), w(s)= The LST of the PDF H(x), R(x), V(x), W(x)
h_k, r_k, v_k , w_k=the kth order moments of the PDF H(x), R(x), V(x), W(x)
M(t)=system size at time t
E(t)= 0 if server is operative at t
=1 if the server is down
S(t)= 0 if the server is free at t
=1 if it is busy
=2 if it is on vacation

ξ(t)=the remaining retrial time if
E(t)=S(t)=0
=the remaining service time if
E(t)=0, S(t)=1
=the remaining repair time if
E(t)=0, S(t)=2

Q(z)= $\lim_{t\to\infty} E(z^{M(t)})$
IR^+=the set of non negative real numbers

3. ANALYSIS OF THE MODEL:

First, we develop some analytical properties of the system under study.

3.1.Fundamental Process.

The process $\zeta(t)=\{E(t),C(t),M(t),\xi(t)\}$ is a Markov process defined on the state space **E**={0,1}×{0,1,2}×I{0,1,2…}×IR^+\{y: y=(0,0,x), x≥0} which can be studied by using a method similar to that of [Aissani,2000]. Thus we restrict analysis to the description of obtained results. First we have that a condition for the system to be stable is

$$\rho=\frac{\lambda}{\delta}(g_1-1)+\frac{\lambda g_1[\theta w_1+(\delta+\lambda)m_1]-\theta}{\delta}<1$$

where $\delta=\dfrac{(\lambda+\theta)r(\lambda+\theta)}{1-r(\lambda+\theta)}$

and [Aissani & Artalejo,1998]:

$$m_1=(1-h(\theta))(w_1+\theta^{-1});$$

It is not surprising that this condition depends on reliability parameters. However, we note that it is independent of the vacation parameter and contrary to the case of linear retrial rate (see [Aissani and Artalejo,1998]) it depends on retrial time distribution. We assume from now that ρ<1, so the stationary probabilities:

$$P_{ij}(m,x)=\lim_{t\to\infty} P\{E(t)=i,C(t)=j,M(t)=m;\xi(t)<x\}$$

exists. Now, define by $Q_{ij}(z,x)$ the PGF in m, and the Laplace transform $f_{ij}(z,s)$ in x . By usual way, we obtain:

$$f_{00}(z,s)=\frac{(\lambda+\theta)[r(\lambda+\theta)-r(s)]}{s(s-\lambda+\theta)[1-r(\lambda+\theta)]}Q_{00}(z,\infty)$$

$$f_{10}(z,s)=\theta Q_{00}(z,\infty)\frac{w(\varepsilon(z))-w(s)}{s(s-\varepsilon(z))}$$

$$f_{01}(z,s)=\frac{\nu+\lambda G(z)}{z}\frac{h(\varepsilon(z))-h(s)}{s(s-\varepsilon(z))}Q_{00}(z,\infty)$$

$$f_{02}(z,s)=\alpha\frac{v(\varepsilon(z))-v(s)}{s(s-\varepsilon(z))}$$ where

$$\alpha=(1-\rho)\frac{\delta}{(\lambda+\delta)v_1}\ ,\ \varepsilon(z)=\lambda-\lambda G(z).$$

3.2. System Size Distribution:

The PDF of the number of customers in the system at an arbitrary point can be derived from previous section as in [Aissani, 2000]:

$$Q(z)=(1-\rho)\frac{r(\lambda+\theta)}{v_1}\frac{(1-z)[1-v(\varepsilon(z))]}{\varepsilon(z)}\times$$

$$\frac{\left(\lambda G(z)+\delta\right)h\left(\varepsilon(z)+\theta w(\varepsilon(z)\right)}{\left[(\lambda G(z)+\delta)h\left(\varepsilon(z)\right)+\theta w(\varepsilon(z))-\left(\lambda+\delta+\theta\right)z\right]}$$

3.3. Decomposition Result:

Using the above formula, we can obtain an interesting decomposition result. More precisely, the number of customers in our model can be expressed as a sum of three independent random variables representing the number of customers in: (i) an unreliable system with FIFO queue without vacation; (ii) our model given that the server is on vacation; (iii) an unreliable retrial queue without vacation given that the server is idle. Such a result is useful when computing higher order moments.

3.4. Reliability and Service Metrics:

Since the breakdown and repair processes are independent of the servicing processes, then the hardware reliability and availability metrics are defined in the usual way. Next, we can define:

Servicing availability: It is the probability that the server is available (in the hardware sense) and free of customers: $p_{00.}=Q_{00}(1,\infty)=\dfrac{\lambda g_1}{\lambda+\delta+\theta}$.

Probability that the server is available and busy by the service of a customer

$$p_{01}=Q_{01}(1,\infty)=\lambda g_1 h_1\frac{\lambda+\delta}{\lambda+\delta+\theta}$$

Probability that the server is on vacation: $p_{02.}=Q_{02}(1,\infty)=(1-\rho)r(\lambda+\theta)$

Average number of customers in the system: This characteristic can be obtained directly using formula of §3.2.:

$$\mathbf{E}\{M\}=\frac{\lambda g_1 v_2}{v_1}+\Psi,$$

where $\Psi=\dfrac{\beta\delta+\gamma}{2\delta(1-\rho)}+\dfrac{\lambda g_1 h_1\delta+\alpha}{\lambda+\theta+\delta}$

$\alpha=\lambda g_1+\lambda^2 g_1 h_1+\theta\lambda g_1 w_1$

$\beta=\lambda g_2 h_1+(\lambda g_1)^2 h_2$

$\gamma=\lambda g_2+2(\lambda g_1)^2 h_1+\lambda\alpha+\theta\lambda g_1 w_1+\theta(\lambda g_1)^2 w_2$

Mean waiting time: From Little's formula we have:

$$\mathbf{E}(W)=\lambda g_1\mathbf{E}(M).$$

4. CONTROL OF VACATION POLICY:

This section illustrates usefulness of the results of previous sections by giving applications to optimal control of the vacation policy.

4.1. Cost Function: Let us consider the following costs.

C_S=setup cost per cycle (each time the server is reopened).
C_h=holding cost per unit time (incurred for each customer present in the system.
C_d=breakdown cost per unit time for a failed server.
C_0=cost per unit time for keeping the server on and in operation.

As usual, the expected exploitation time costs per unit can be expressed as

$$C=\frac{C_S}{E(L)}+C_h E(M)+C_0\frac{E(A)}{E(L)}+C_d\frac{E(B)}{E(L)}$$

where A and B are the sojourn times in the corresponding states during a cycle L with mean $\mathbf{E}(L)=\dfrac{E(V)}{(1-\rho)r(\lambda+\theta)}$. We consider the policy under which the server is turned off when system becomes empty and it is turned on again when the number of customers reaches the threshold N. In this case, the cost function C(N) is expressed as

$$C(N)=\frac{C_S\lambda g_1(1-\rho)\delta}{N(\lambda+\theta+\delta)}+C_h\left\{\frac{N-1}{2}+\Psi\right\}$$

up to a fixed cost which is independent of N.

4.2. Optimal Threshold. We are now able to find the optimal value N* which minimizes the cost function C(N) . Since this cost is a convex function, then the optimal value is one of integers adjacent to the value

$$N^*=\sqrt{\frac{2C_S\lambda g_1(1-\rho)r(\lambda+\theta)}{C_h}}.$$

4.3. Effect of Retrials Upon The 0ptimal N*.

We consider here effect of retrial distribution R(.) upon the optimal values for both vacation policies. Consider the class $\Im_m^\sigma$ of PDF with mean m and finite variance σ^2 , and $\Im_m^{NBUE}$ the class of all PDF on $[0,\infty)$ with mean m that are **NBUE.** Recall that a PDF on IR^+ is **NBUE** if and only if $\int_x^\infty \bar{F}(y)dy\le m\bar{F}(x)$ for $x\ge0$. Let $\theta_m=0$ if $x<m$, and $\theta_m=1$ if $x\ge m$.

(1) If retrial time distribution R(x) belongs to the class $\Im_m^\sigma$, then the optimal value of N* is bounded

as follows: $N^*_L<N^*<N^*_U$ where upper and lower bounds are given respectively by

$$N^*_L = \theta_0(\eta)\sqrt{\frac{2S\lambda g_1(\eta)}{h}}$$

$$\eta = e^{-\lambda r_1} g_1 - (g_1 - 1) - \lambda g_1 h_1$$

$$N^*_U = \theta_0(\chi)\sqrt{\frac{2S\lambda g_1(\chi)}{h}}$$

$$\chi = r_U(\lambda) g_1 - (g_1 - 1) - \lambda g_1 h_1$$

and $r_U(\lambda) = \dfrac{r_2 - r_1^2}{r_2} + \dfrac{r_1^2}{r_2} e^{-\lambda\left(r_1 + \frac{r_2 - r_1^2}{r_1^2}\right)}$.

(2) If R(x) belong to $\Im_m^{NBUE}$ then $N^*_L<N^*_{NBUE}<N^*_{EXP}$ where N^*_{NBUE} is the optimal value for an $M^X/G/1$ vacation queue with **NBUE** retrial time distribution, and N^*_{Exp} is the optimal value for the model with vacation and constant retrials [Artalejo, 1997].

Remark. For sake of space, we have considered here only the case of a reliable server (θ=0). The first inequality gives approximations (in fact lower and upper bounds) on the optimal threshold N* when the retrial time distribution is unknown, but we have a partial information about the first two moments. The second one tell us about the case when the partial information concerns an ageing class of retrial time distribution.

5. CONTROL OF RETRIAL POLICY.

We now investigate the problem of optimal control of retrial parameter when the system operates under the N-policy. Note that in fact the cost function depends only on the real value δ, and not on the concrete aspect of the retrial time distribution. So, we consider the problem of choice of an optimal value δ which minimizes the cost function C. Differentiate wrt: δ gives $\dfrac{C_S \lambda g_1}{N C_h} = F(\delta)$

where $$F(\delta) = 1 - \frac{1}{2}\left(\frac{\lambda+\theta+\delta}{\delta(1-\rho)}\right)^2 \times$$

$$\times \frac{(\lambda+\theta-\alpha)\beta - \gamma(1-\lambda g_1 h_1)}{\alpha - (\lambda+\theta)\lambda g_1 h_1}$$

The function F(δ) satisfies the following properties:

(i) Its domain's value is F: $[\delta_e, +\infty) \to IR^+$ where $\delta_e = \dfrac{\alpha - \lambda - \theta}{1 - \lambda g_1 h_1}$

(ii) $\lim_{\delta \to \delta_e} F(\delta) = +\infty$

(iii) $\lim_{\delta \to +\infty} F(\delta) = \Pi$

where

$$\Pi = \frac{1}{1-\lambda g_1 h_1} \times$$
$$\times \left(1 + \frac{(1-\lambda g_1 h_1)(\gamma - 2\lambda g_1 h_1 \xi) - (\lambda+\theta-\alpha)\beta}{2\xi(1-\lambda g_1 h_1}\right)$$

Note finally that $C'(\delta)<0$ if and only if $F(\delta)>C_S\lambda g_1/C_h N$. So the optimal value of δ* is:

(i) If $\dfrac{C_S\lambda}{C_h N} \le \Pi$, then $\delta^* = +\infty$.

(ii) If $\dfrac{C_S\lambda}{C_h N} > \Pi$, then δ* is solution of the equation $F(\delta) = \dfrac{C_S \lambda g_1}{C_h N}$, for $\delta>\delta_e$.

6.NUMERICAL ILLUSTRATIONS:

In this section we illustrate the effect of parameters (retrial, vacation and breakdowns) on system performances. In the remainder of this section we take the basic data of [Artalejo, 1997]: $\lambda=1$, $g_1=1$, $g_2=0$, $h_1=0.25$, $h_2=1$. Concerning the maintenance parameters we take $w_1=0.1$ and $w_2=1$.

First, we show effect of failure rate on the retrial parameterδ. In figure 1 we have plotted the function δ(θ) for different retrial PDF with mean $r_1=1$:

(i) Hyperexponential (H_2).
(ii) Exponential (Exp):
(iii) Determinist (D):

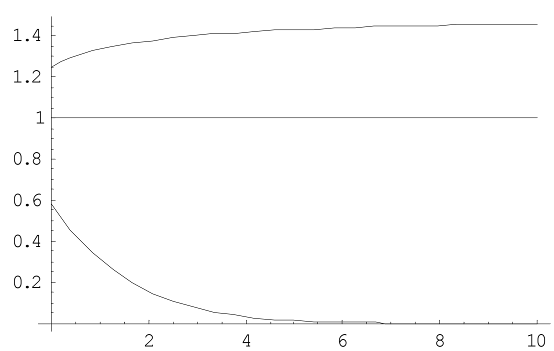

Figure 1. Effect of failure rate θ on δ.

We observe that the parameter δ increases in the case (i) and decreases in the case (iii) as the failure rate increases. In the case (ii) the parameter δ is independent of the failure rate. This can be easily understood from exponential nature of retrial time.

Figure 2 plots expectation E(M) versus failure rate θ and ratio v_2/v_1. We note that E(M) decreases when θ and v_2/v_1 increases and increases otherwise.

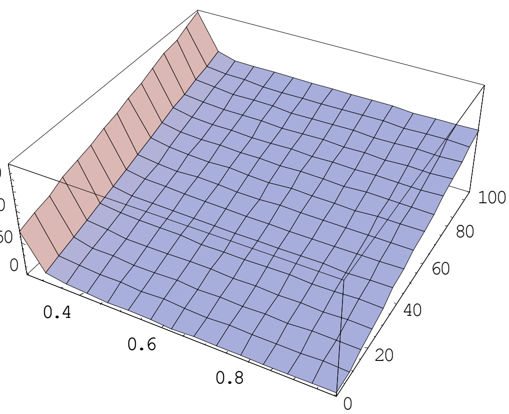

Figure 2. Effect of breakdowns and vacations on Mean system size .

Figure 3 shows effect of failure rate on the optimal threshold for different values of C_S/C_h=10, 50 and 100. We have considered a 2-Erlangian retrial distribution (E_2) with mean r_1=0.5; We note that the optimal threshold increases with the ratio C_S/C_h.

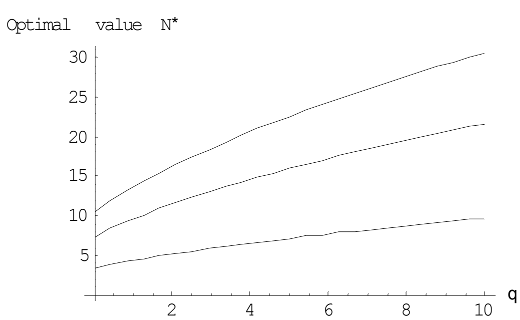

Figure 3. Effect of θ on the optimal N*.

Table 1 compares lower and upper bounds on the optimal value N* for different parametric (Exp, D, H_2) and non parametric (NBUE) retrial PDF which typify some PDF observed in practice. For each of these choices we varied the ratio C_S/C_h from 0.5 to 10^5.

Figure 4 illustrates behaviour of the bounds as a function of the mean retrial time for different values of C_S/C_h=10, 1, 0.1 . For a given value of this ratio, the dot-dashed curve corresponds to a lower bound and the continuous curve to an upper bound. The lowest pair of curves corresponds to the case C_S/C_h=0.1 . We see that lower bound tends to be more closed to the upper bound curve for small values of r_1 and C_S/C_h.

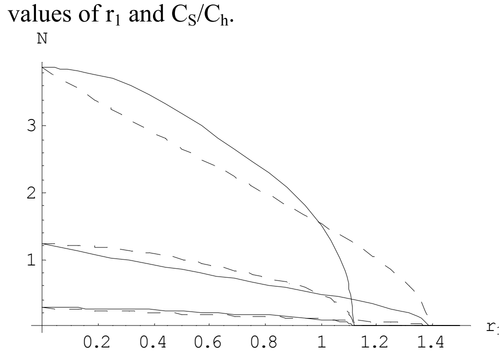

Figure 4.Bounds on the optimal threshold.

Finally, table 2 shows the joint effect of retrials and breakdowns upon the optimal value N* and its corresponding minimum expected cost. The optimal value N* increases and the cost decreases when both δ and θ increases (see also figure 5).

7.CONCLUSION: In this work we studied the effect of retrials, vacations and breakdowns on the performance metrics of queueing service systems. We have showed how to control the vacation and retrial mechanisms. A similar study can be provided to control the maintenance actions.

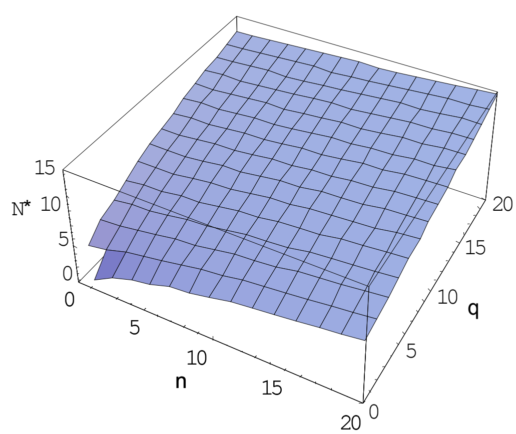

Figure 5. Effect of retrial rate δ and failure rate on the optimal threshold N*.

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Table 1. Lower and Upper bounds on the optimal value N*

S/h	0.5	1	10	24	10^5
Lower bound Determinist retrials	0.18708	0.37416	0.83666	1.29614	83.666
Exponential					
m=2; σ^2=4	0.48370	0.68318	2.16023	3.3466	216.02314
m=1; σ^2=1	0.63245	0.894427	2.82842	4.38178	282.8427
2-Erlang					
m=2; σ^2=4	0.2626	0.3714	1.1747	1.8198	117.473
m=2; σ^2=2	0.38729	0.54772	1.732	2.68328	173.205
NBUE					
m=2; σ^2=1	0.32989	0.46654	1.47523	2.28525	147.533
Upper bound					
m=2; σ^2=4	0.64800	0.916500	2.8982	4.48998	289.8275
m=1; σ^2=1	0.67820	0.959100	3.0331	4.6989	303.3150

Table 2. Optimal Thresholds N* and its corresponding minimum cost.
$C_S=5, C_h=1,\lambda=1,g_1=1,g_2=0,w_1=0.1,w_2=1.$

θ=0; $\delta\rightarrow$	0.35	0.4	0.5	1	10	20	50	∞
ρ	0.9642	0.875	0.75	0.5	0.275	0.2625	0.255	0.25
N*	0.3042	0.5976	0.9128	1.5811	2.5672	2.6502	2.7025	2.7386
C(N*)	7.5223	6.4904	5.3295	3.8311	3.1878	3.1705	3.1611	3.1552
θ=0.5;$\delta\rightarrow$	0.85	1	10	20				∞
ρ	0.01	0.05	0.23	0.24				0.25
N*	1.887	1.9493	2.5876	2.6589				2.7386
C(N*)	3.9636	3.8340	3.2075	3.1804				3.1552
θ=1;$\delta\rightarrow$	5	10	20	50	100			∞
ρ	0.12	0.18	0.185	0.23	0.24			0.25
N*	2.5070	2.6060	2.6671	2.7086	2.7233			2.738
C(N*)	3.2988	3.2256	3.1898	3.1689	3.1820			3.1552
θ=10;$\delta\rightarrow$	50	100						∞
ρ	0.03	0.16						0.25
N*	2.7564	2.7482						2.738
C(N*)	3.2457	3.1946						3.1552