# A GENERALIZED MARKOVIAN QUEUE TO MODEL AN OPTICAL PACKET SWITCHING MULTIPLEXER 

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#### Abstract

Packet and burst switching have been proposed for optical networks because they can better accommodate bursty traffic generated by IP applications. In optical packet switching networks the payload and the header of the same packet are conveyed in the same channel, while burst switching networks allow the separate transportation of the payload and the header of the same burst. In this paper we consider an optical packet switching node that assigns arriving packets to channels in a link with $c$ available data channels (wavelengths) and a buffer of $L-c$ size. The paper applies the novel MM $\sum_{k=1}^{K} C P P_{k}$ $/ \mathrm{GE} / \mathrm{c} / \mathrm{L}$ G-queue to model optical packet switching nodes. It is worth emphasizing that our method can be applied to model burst switching nodes as well. Moreover, we show that a model previously presented in the literature is only the special case of our model. Numerical results quantitatively demonstrate that the characteristics (e.g.: burstiness) of the offered traffic have a significant impact on the performance of optical nodes.


Keywords: optical packet switching, optical burst switching, MM $\sum_{k=1}^{K} C P P_{k} / \mathrm{GE} / \mathrm{c} / \mathrm{L}$ G-queue, G-networks

## 1 Introduction

To efficiently accommodate bursty IP data traffic two technical solutions (packet and burst switching) are being proposed for networks based on optical technology. The final aim is to have networks that switch packets of constant or variable length while the payload data stays in the optical domain. In burst switching networks payload data and its control data (header) are transported in different channels, while packet switching networks convey payload data and its header in the same channel [El-Bawab and Shin, 2002, Yao et al., 2002].

In this paper we develop a new model for optical nodes operating in either optical packet switching or burst switching networks. To evaluate the performance of optical nodes a decomposition approach is used. Namely, the performance of an optical node is determined if we can evaluate the performance of multiplexers before the transmission links. That is, we consider an optical packet (or burst) switching multiplexer that assigns arriving packets (or bursts) to $c$ available data channels (wavelengths) and has a buffer for $L-c$ packets (or bursts). Therefore, we propose the use of the MM $\sum_{\mathbf{k}=1}^{\mathrm{K}} C P P_{k} / \mathrm{GE} / \mathrm{c} / \mathrm{L}$ G-queue to model nodes in both kinds of networks (burst and packet switching), which queue has been proposed recently in [Chakka et al., 2003]. This is a homogeneous multiserver queue with $c$ servers, GE service times and with $K$ independent customer arrival streams, each of which is a CPP, i.e. a Poisson point process with batch arrivals of geometrically distributed batch size.

The use of the MM $\sum_{k=1}^{K} C P P_{k}$ process to model packet or burst arrival process is motivated by the following reason. Recent studies have shown that the traffic in today's telecommunications systems often exhibits burstiness - i.e. batches of transmission units (e.g. packets) arrive together - and correlation among interarrival times. As a consequence different mod-
els have been proposed. These models include the compound Poisson process ( CPP ) in which the interarrival times are assumed to have generalized exponential (GE) probability distribution [Kouvatsos, 1994], the Markov modulated Poisson process (MMPP) and self-similar traffic models such as Fractional Brownian Motion (FBM) [Mandelbrot and Ness, 1968, Norros, 1994]. A CPP traffic model often gives a good representation of burstiness of the traffic from one or more sources, e.g. [Bhabuta and Harrison, 1997, Fretwell and Kouvatsos, 1999], but not of the autocorrelations observed in real traffic. Conversely, the MMPP models can capture auto-correlation but not burstiness, e.g. [Fretwell and Kouvatsos, 1997, Meier-Hellstern, 1989]. The self-similar models such as FBM can account for both auto-correlation and burstiness, but they are analytically intractable in a queueing context. Often, the traffic to a node is the superposition of traffic from a number of sources complicating the system analysis further. The MM $\sum_{\mathbf{k}=1}^{\mathbf{K}} C P P_{k}$ captures the burstiness and correlation of the traffic, and its parameter $K$ can be used to model various traffic passing optical nodes from different sources in a flexible manner. Moreover, the Markov modulated $\sum_{k=1}^{K} C P P_{k} / \mathrm{GE} / \mathrm{c} / \mathrm{L}$ Gqueue is mathematically tractable with efficient analytical solution for the steady state probabilities with the use of mathematically oriented transformations [Chakka et al., 2003]. To obtain the steady state probabilities and thus the performance measures either the spectral expansion method [Chakka, 1995] or Naoumov's method [Naoumov et al., 1997] extended for QBD processes, or the matrix-geometric solution method [Neuts, 1995] can be used.

Related to the performance analysis aspect, Turner has proposed a birth-death process to analyze a multiplexer in optical burst switched networks [Turner, 1999]. However, Turner's model has some limitations like the as-
sumption of exponential burst arrival process, exponential service times and constant burst size. It can be shown and numerically demonstrated that Turner's model is the special case of our model. Moreover, our model overcomes the limitations of Turner's model as regards the arrival process.

The rest of the paper is organized as follows. The proposed model is described in Section 2. Some numerical results are then presented in Section 3. The paper concludes in Section 4.

## 2 Model description

Since we consider a multiplexer before a transmission link with $c$ available data channels (wavelengths) and a buffer for $L-c$ packets (or bursts), a queueing model for a multiplexer has $c$ servers and $L$ queueing capacity ${ }^{1}$ for packets (or bursts). In what follows we outline the important characteristics of the proposed model.

### 2.1 The Arrival Process

The arrival and service processes are modulated by the same continuous time, irreducible Markov phase process with $N$ states. Let $Q$ be the generator matrix of this process, given by

$$
Q=\left[\begin{array}{cccc}
-q_{1} & q_{1,2} & \ldots & q_{1, N} \\
q_{2,1} & -q_{2} & \ldots & q_{2, N} \\
\vdots & \vdots & \ddots & \vdots \\
q_{N, 1} & q_{N, 2} & \ldots & -q_{N}
\end{array}\right]
$$

where $q_{i, k}(i \neq k)$ is the instantaneous transition rate from phase $i$ to phase $k$, and

$$
q_{i}=\sum_{j=1}^{N} q_{i, j}, \quad q_{i, i}=0 \quad(i=1, \ldots, N)
$$

Let $\mathbf{r}=\left(r_{1}, r_{2}, \ldots, r_{N}\right)$ be the vector of equilibrium probabilities of the modulating phases. Then, $\mathbf{r}$ is uniquely determined by the equations:

$$
\mathbf{r} Q=0 \quad ; \quad \mathbf{r} \mathbf{e}_{N}=1
$$

where $\mathbf{e}_{N}$ stands for the column vector with $N$ elements, each of which is unity.

The arrival process ( $\mathrm{MM} \sum_{k=1}^{K} C P P_{k}$ ) is the superposition of $K$ independent CPP arrival streams of customers ${ }^{2}$, in a Markov modulated environment. The customers of different arrival streams are not distinguishable. The parameters of the GE inter-arrival time distribution of the $k^{t h}(1 \leq k \leq K)$ customer arrival stream in phase $i$ are $\left(\sigma_{i, k}, \theta_{i, k}\right)$. Thus, all the $K$ arrival pointprocesses are Poisson, with batches arriving at each

[^0]point having geometric size distribution. Specifically, the probability that a batch is of size $s$ is $\left(1-\theta_{i, k}\right) \theta_{i, k}^{s-1}$, in phase $i$, for the $k^{t h}$ stream of customers.

Let $\sigma_{i, .}, \overline{\sigma_{i, .}}$ be the average arrival rate of customer batches and customers in phase $i$ respectively. Let $\sigma, \bar{\sigma}$ be the overall average arrival rate of batches and customers respectively. Then,

$$
\begin{align*}
\sigma_{i, .}=\sum_{k=1}^{K} \sigma_{i, k} & ; \quad \overline{\sigma_{i, .}}=\sum_{k=1}^{K} \frac{\sigma_{i, k}}{\left(1-\theta_{i, k}\right)}  \tag{1}\\
\sigma=\sum_{i=1}^{N} \sigma_{i, .} r_{i} & ; \quad \bar{\sigma}=\sum_{i=1}^{N} \overline{\sigma_{i, .}} r_{i}
\end{align*}
$$

Because of the superposition of many CPP's, the overall arrivals in phase $i$ can be considered as bulk-Poisson ( $M^{[x]}$ ) with arrival rate $\sigma_{i, .}$ and with a batch size distribution $\left\{\pi_{l / i}\right\}$ (the probability of batch size being $l$ given that the phase is $i$ ) that is more general than mere geometric. The probability that this batch size is $l$ is given by,

$$
\begin{align*}
\pi_{l / i}= & \sum_{k=1}^{K} \frac{\sigma_{i, k}}{\sigma_{i, .}}\left(1-\theta_{i, k}\right) \theta_{i, k}^{l-1}  \tag{2}\\
& \sum_{l=1}^{\infty} \pi_{l / i}=1.0 \tag{3}
\end{align*}
$$

The overall batch size distribution is then given by,

$$
\begin{equation*}
\pi_{l / .}=\sum_{i=1}^{N} r_{i} \pi_{l / i} \tag{4}
\end{equation*}
$$

Define $\pi_{i, l}$ as the probability that a given batch arrival is during phase $i$ and is of size $l$, then $\pi_{i, l}=r_{i} \pi_{l / i}$.

### 2.2 The GE Multi-server

Each data channel will be modelled as a server. Therefore there are $c$ homogeneous servers in parallel, each with GE-distributed service times with parameters ( $\mu_{i}, \phi_{i}$ ) in phase $i$. The service discipline is FCFS and each server serves at most one customer at any given time. The operation of the GE server is similar to that described for the CPP arrival processes above. $L$ denotes the queueing capacity in all phases, including the packets in service, if any. $L$ can be finite or infinite. When the number of packets is $j$ and the arriving batch size of customers is greater than $L-j$ (assuming a finite $L$ ), we assume that only $L-j$ customers are taken in and the rest are rejected.

However, the batch size associated with a service completion is bounded by one more than the number of customers waiting to commence service at the departure instant. For queues of length $c \leq j<L+1$ (including any packets in service), the maximum batch size at a departure instant is $j-c+1$, only one server being
able to complete a service period at any one instant under the assumption of exponentially distributed batchservice times. Thus, the probability that a departing batch has size $s$ is $\left(1-\phi_{i}\right) \phi_{i}^{s-1}$ for $1 \leq s \leq j-c$ and $\phi_{i}^{j-c}$ for $s=j-c+1$. In particular, when $j=c$, the departing batch has size 1 with probability one, and this is also the case for all $1 \leq j \leq c$ since each packet is already engaged by a server and there are then no packets waiting to commence service.

It is assumed that the first packet in a batch arriving at an instant when the queue length is less than $c$ (so that at least one server is free) never skips service, i.e. always has an exponentially distributed service time. However, even without this assumption the methodology described in this paper is still applicable.

### 2.3 Negative Customers

The parameters of the GE inter-arrival time distribution of negative customers are $\left(\rho_{i}, \delta_{i}\right)$ in phase $i$. That is, the inter-arrival time probability distribution function is $1-\left(1-\delta_{i}\right) e^{-\rho_{i} t}$ for the negative customers in phase $i$. Thus, the negative customer arrival point-process is Poisson, with batches arriving at each point having geometric size distribution.

A negative customer removes a positive customer in the queue, according to a specified killing discipline. When a batch of negative customers of size $l(1 \leq l<j-$ c) arrives, $l$ positive customers are removed from the end of the queue leaving the remaining $j-l$ positive customers in the system. If $l \geq j-c \geq 1$, then $j-c$ positive customers are removed, leaving none waiting to commence service (queue length equals to $c$ ). If $j \leq c$, the negative arrivals have no effect.
$\overline{\rho_{i}}$, the average arrival rate of negative customers in phase $i$ and $\bar{\rho}$, the overall average arrival rate of negative customers are given by,

$$
\begin{equation*}
\overline{\rho_{i}}=\frac{\rho_{i}}{1-\delta_{i}} \quad ; \quad \bar{\rho}=\sum_{i=1}^{N} r_{i} \overline{\rho_{i}} \tag{5}
\end{equation*}
$$

Negative customers remove (positive) customers in the queue and have been used to model random neural networks, task termination in speculative parallelism, faulty components in manufacturing systems and server breakdowns [Fourneau et al., 1996, Fourneau and Hernandez, 1993]. The name G-queue has been adopted for queues with negative customers in acknowledgement of Gelenbe who first introduced them. This queueing model can account for burstiness and correlation, but in addition the negative customers, with an appropriate killing discipline, can represent additional behaviours such as breakdowns, killing signals, cell losses and load balancing. We show in Section 3 how negative customers can be used to model packet losses.

### 2.4 Condition for Stability

When $L$ is finite, the system is ergodic since the representing Markov process is irreducible. Otherwise, i.e. when $L=\infty$, the overall average departure rate increases with the queue length, and its maximum (the overall average departure rate when the queue length tends to $\infty$ ) can be determined as,

$$
\begin{equation*}
\bar{\mu}=c \sum_{i=1}^{N} \frac{r_{i} \mu_{i}}{1-\phi_{i}} \tag{6}
\end{equation*}
$$

Hence, we conjecture that the necessary and sufficient condition for the existence of steady state probabilities is

$$
\begin{equation*}
\bar{\sigma}<\bar{\rho}+\bar{\mu} . \tag{7}
\end{equation*}
$$

### 2.5 The Steady State Balance Equations

The state of the system at any time $t$ can be specified completely by two integer-valued random variables, $I(t)$ and $J(t)$. $I(t)$ varies from 1 to $N$, representing the phase of the modulating Markov chain, and $0 \leq J(t)<L+1$ represents the number of positive customers in the system at time $t$, including any in service. The system is now modelled by a continuous time discrete state Markov process, $\bar{Y}$ ( $Y$ if $L$ is infinite), on a rectangular lattice strip. Let $I(t)$, the phase, vary in the horizontal direction and $J(t)$, the queue length or level, in the vertical direction. We denote the steady state probabilities by $\left\{p_{i, j}\right\}$, where $p_{i, j}=\lim _{t \rightarrow \infty} \operatorname{Prob}(I(t)=i, J(t)=j)$, and let $\mathbf{v}_{j}=$ $\left(p_{1, j}, \ldots, p_{N, j}\right)$.

The process $\bar{Y}$ evolves due to the following instantaneous transition rates:
(a) $q_{i, k}$ - purely lateral transition rate - from state $(i, j)$ to state $(k, j)$, for all $j \geq 0$ and $1 \leq i, k \leq$ $N \quad(i \neq k)$, caused by a phase transition in the Markov chain governing the arrival phase process;
(b) $B_{i, j, j+s}-s$-step upward transition rate - from state $(i, j)$ to state $(i, j+s)$, for all phases $i$, caused by a new batch arrival of size $s$ customers. For a given $j, s$ can be seen as bounded when $L$ is finite and unbounded when $L$ is infinite;
(c) $C_{i, j, j-s}-s$-step downward transition rate - from state $(i, j)$ to state $(i, j-s),(j-s \geq c+1)$ for all phases $i$, caused by a batch service completion of size $s$, or a batch arrival of negative customers of size $s$;
(d) $C_{i, c+s, c}-s$-step downward transition rate - from state $(i, c+s)$ to state $(i, c)$, for all phases $i$, caused by a batch arrival of negative customers of size $\geq s$ or a batch service completion of size $s(1 \leq s \leq L-c)$;
(e) $C_{i, c-1+s, c-1}-s$-step downward transition rate, from state $(i, c-1+s)$ to state $(i, c-1)$, for all phases $i$, caused by a batch departure of size $s(1 \leq s \leq L-c+1) ;$
(f) $C_{i, j+1, j}-1$-step downward transition rate, from state $(i, j+1)$ to state $(i, j),(c \geq 2 ; 0 \leq j \leq c-$ 2 ), for all phases $i$, caused by a single departure.

Define,

$$
\begin{aligned}
B_{j-s, j}= & \operatorname{Diag}\left[B_{1, j-s, j}, B_{2, j-s, j}, \ldots, B_{N, j-s, j}\right] \\
& (j-s<j \leq L) ; \\
B_{s}= & B_{j-s, j} \quad(j<L) \\
= & \operatorname{Diag}\left[\ldots, \sum_{k=1}^{K} \sigma_{i, k}\left(1-\theta_{i, k}\right) \theta_{i, k}^{s-1}, \ldots\right] ; \\
\Sigma_{k}= & \operatorname{Diag}\left[\sigma_{1, k}, \sigma_{2, k}, \ldots, \sigma_{N, k}\right] \\
& (k=1,2, \ldots, K) ; \\
\Theta_{k}= & \operatorname{Diag}\left[\theta_{1, k}, \theta_{2, k}, \ldots, \theta_{N, k}\right] \\
& (k=1,2, \ldots, K) ; \\
\Sigma= & \sum_{k=1}^{K} \Sigma_{k} ; \\
R= & \operatorname{Diag}\left[\rho_{1}, \rho_{2}, \ldots, \rho_{N}\right] ; \\
\Delta= & \operatorname{Diag}\left[\delta_{1}, \delta_{2}, \ldots, \delta_{N}\right] ; \\
M= & \operatorname{Diag}\left[\mu_{1}, \mu_{2}, \ldots, \mu_{N}\right] ; \\
\Phi= & \operatorname{Diag}\left[\phi_{1}, \phi_{2}, \ldots, \phi_{N}\right] ; \\
C_{j}= & j M \\
= & c M=C \\
C_{j+s, j}= & \operatorname{Diag}\left[C_{1, j+s, j}, C_{2, j+s, j}, \ldots, C_{N, j+s, j}\right] ; \\
E= & \operatorname{Diag}\left(\mathbf{e}_{N}^{\prime}\right) .
\end{aligned}
$$

Then, we get,

$$
\begin{aligned}
B_{s}= & \sum_{k=1}^{K} \Theta_{k}^{s-1}\left(E-\Theta_{k}\right) \Sigma_{k} ; \\
B_{1}= & B=\sum_{k=1}^{K}\left(E-\Theta_{k}\right) \Sigma_{k} ; \\
B_{L-s, L}= & \sum_{k=1}^{K} \Theta_{k}^{s-1} \Sigma_{k} ; \\
C_{j+s, j}= & C(E-\Phi) \Phi^{s-1}+R(E-\Delta) \Delta^{s-1} \\
& (c+1 \leq j \leq L-1 ; s=1,2, \ldots, L-j) ; \\
= & C(E-\Phi) \Phi^{s-1}+R \Delta^{s-1} \\
& (j=c ; s=1,2, \ldots, L-c) ; \\
= & C \Phi^{s-1} \\
& (j=c-1 ; s=1,2, \ldots, L-c+1) ; \\
= & 0(c \geq 2 ; 0 \leq j \leq c-2 ; s \geq 2) ; \\
= & C_{j+1}(c \geq 2 ; 0 \leq j \leq c-2 ; s=1) .
\end{aligned}
$$

The steady state balance equations are,
(1) For the $L^{t h}$ row or level:

$$
\begin{equation*}
\sum_{s=1}^{L} \mathbf{v}_{L-s} B_{L-s, L}+\mathbf{v}_{L}[Q-C-R]=0 \tag{8}
\end{equation*}
$$

(2) For the $j^{\text {th }}$ row or level:

$$
\begin{align*}
& \sum_{s=1}^{j} \mathbf{v}_{j-s} B_{s}+\mathbf{v}_{j}\left[Q-\Sigma-C_{j}-R I_{j>c}\right]+ \\
& \sum_{s=1}^{L-j} \mathbf{v}_{j+s} C_{j+s, j}=0 \quad(0 \leq j \leq L-1) ; \tag{9}
\end{align*}
$$

(3) Normalization

$$
\begin{equation*}
\sum_{j=0}^{L} \mathbf{v}_{j} \mathbf{e}_{N}=1 \tag{10}
\end{equation*}
$$

where, $I_{j>c}=1$ if $j>c$ else 0 , and $\mathbf{e}_{N}$ is a column vector of size $N$ with all ones.

Each equation $((8,9,10))$ has all the unknown vectors $\mathbf{v}_{\mathbf{j}}$ 's. If $L$ is unbounded, then each of these are infinite number of equations in infinite number of unknowns, $\mathbf{v}_{\mathbf{j}}$ 's, and each equation is infinitely long containing all the infinite number of unknowns. Also, the coefficient matrices of $\mathbf{v}_{\mathbf{j}}$ are $\mathbf{j}$-dependent. It may be noted that there has been neither a solution nor a solution methodology to solve these equations. In [Chakka et al., 2003], a novel methodology is developed to solve these equations exactly and efficiently. First these complicated equations are transformed to a computable form by using certain mathematically oriented transformations. The resulting transformed equations are of the QBD-M type (QBD with simultaneous-multiple-bounded births and simultaneous-multiple-bounded deaths) and hence can be solved by one of the several available methods, viz. the spectral expansion method, Bini-Meini's method or the matrix-geometric method with folding or block size enlargement [Haverkort and A.Ost, 1997].

### 2.6 Performance Measures

Some performance measures can be derived as follows:

- Packet loss probability

$$
\begin{equation*}
\sum_{j=0}^{L} \sum_{l=L-j+1}^{\infty} \mathbf{v}_{j}\left(\pi_{1, l}, \ldots, \pi_{N, l}\right)^{\prime} \frac{l-(L-j)}{l} \tag{11}
\end{equation*}
$$

- Average departure rate of positive customers

$$
\begin{equation*}
\bar{\nu}=\sum_{s=1}^{L-c+1} s \nu_{s} \tag{12}
\end{equation*}
$$

where

$$
\begin{align*}
& \nu_{n}=\sum_{i=1}^{N} \sum_{j=c+n}^{L} p_{i, j}\left(1-\phi_{i}\right) \phi_{i}^{n-1} c \mu_{i}+ \\
& \quad \sum_{i=1}^{N} p_{i, c+n-1} \phi_{i}^{n-1} c \mu_{i}(n=2, \ldots, L-c+1) \\
& \text { and }  \tag{14}\\
& \nu_{1}= \\
& \sum_{i=1}^{N} \sum_{j=1}^{c} p_{i, j} j \mu_{i}+\sum_{i=1}^{N} \sum_{j=c+1}^{L} p_{i, j}\left(1-\phi_{i}\right) c \mu_{i}
\end{align*}
$$

## 3 Numerical results

Three numerical results are presented. First, we show that Turner's model is the special case of our model. Next, we present the impact of bursty traffic on the performance of the system. Note that in the first two cases, no negative customers are allowed in the system. Then, we show how the throughput of connections can be determined through the presence of negative customers.

### 3.1 Turner's Model is the Special Case of our Model

In this section we demonstrate that Turner's model for burst switching is the special case of our model by letting $K=1, N=1,\left[q_{i, j}\right]=[0], \quad \theta_{1,1}=0, \phi_{1}=$ $0, \mu_{1}=1$. It easy to prove that the traffic load is determined by $\sigma_{1,1}$.


Figure 1: Packet loss probability vs load and $c$
Figure 1 is exactly the same as Figure 2 in [Turner, 1999], except that the data was produced by our model with the parameter settings mentioned earlier. In order to demonstrate the equivalence, the results were calculated and compared to 20 significant digits using both models for a subset of the parameter set displayed on Figure 1. The calculations were executed on a Sun Ultra 60 Workstation, which had a machine ep$\operatorname{silon}^{3} \epsilon=1.9 * 10^{-34}$. Table 1 summarizes the outcome. It is clear that the differences between the results produced by the two models are $O(\epsilon)$.

[^1]Table 1: Numerical comparison of Turner's model and the $\mathrm{MM} \sum_{k=1}^{K} C P P_{k} / \mathrm{GE} / \mathrm{c} / \mathrm{L}$ model for $c=32$

| load | number of identical digits |  |  |  |  | exponent of numerical value |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $b$ | 0 | 8 | 16 | 24 | 32 | 0 | 8 | 16 | 24 | 32 |
| 0.2 | 20 | 16 | 9 | 4 | 0 | -13 | -18 | -24 | -30 | -35 |
| 0.3 | 20 | 20 | 16 | 13 | 9 | -9 | -13 | -17 | -21 | -25 |
| 0.4 | 20 | 20 | 20 | 18 | 15 | -6 | -9 | -12 | -16 | -19 |
| 0.5 | 20 | 20 | 20 | 20 | 20 | -4 | -7 | -9 | -12 | -14 |
| $\vdots$ |  |  |  |  |  | $\vdots$ |  |  |  |  |
| 1.2 | 20 | 20 | 20 | 20 | 20 | -1 | -1 | -1 | -1 | -1 |

### 3.2 Impact of Bursty Traffic

In this section we show the impact of the burstiness of the offered traffic on the performance of the multiplexer.


Figure 2: Packet loss probability vs load and $c$
Figure 2 plots the packet discard probability for this numerical example where batch arrivals are allowed. It is clearly observed that batch arrivals have a significant impact on the performance of the system and batch arrivals can be better handled by increasing the buffer space (at the expense of some queueing delay) than by increasing the number of channels. The performance of 256 channels with no buffer is worse than that of 32 channels with a buffer for 8 packets in our example for relative load values above 0.4.

### 3.3 Impact of the Connection Loss on the Connection Throughput

In this section we present an approximation to calculate the performance parameter (throughput) of a connection based in the presented queueing model. We also illustrate, then, the impact of a packet loss on the performance of a connection. The considered problem here is the approximation of the throughput of two communicating peers in optical networks. A preliminary approximation can be proposed as follows. The throughput of two communicating peers can be approximated with the queueing model of a single node incorporating the packet loss phenomena along the path. It is showed based on measurements in [Yajnik et al., 1999] that packet loss can be modelled as a 2-state Markov


Figure 3: Effect of the negative customer arrival process
chain model. Therefore, the MM $\sum_{k=1}^{K} C P P_{k} / \mathrm{GE} / \mathrm{c} / \mathrm{L}$ G-queue can be applied in this case, where negative customers model the loss along the path, and the departure rate of positive customers is the performance measure related to the throughput of a connection.

Figure 3 illustrates the dependency of the customer departure rate on the parameter controlling the packet loss process (modelled by negative customers). It can be observed that the correlation of the packet losses has a significant impact on the performance of the system.

## 4 Conclusions

We have applied a new queueing model for the performance analysis of optical packet switching nodes, which model overcomes some of the limitations of the previous work. Moreover, it is shown that Turner's model is the special case of our model. Numerical results quantitatively demonstrate that the characteristics (e.g.: burstiness) of the offered traffic have a significant impact on the performance of optical nodes. In addition the proposed model is able to handle large or unbounded batch sizes, both in arrivals and services, with great computational efficiency and hence may have definite advantages over BMAP based models.

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[^0]:    ${ }^{1}$ Including the packets (or bursts) in service.
    ${ }^{2} \mathrm{~A}$ customer denotes either a packet or a burst

[^1]:    ${ }^{3}$ The machine epsilon is the smallest floating point number that bounds the roundoff in individual floating point operations.

