Metamodels for real-time control - an automotive design study.

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Abstract— **This paper examines the use of *metamodels* in the context of rapid prototyping for real-time control systems. It is desired that a drive by wire throttle controller be designed to minimise the acceleration oscillations which are a result of the first torsional mode of automotive drivelines. A response surface metamodel is fitted to the output of a complex experimentally verified model of the vehicle and driveline. Subsequently, the metamodel is used as a rapid prototyping tool, and as a basis for a final closed-loop design by evolutionary methods.**

Keywords— **Response surface methodology, metamodelling, evolutionary optimisation, rapid prototyping.**

I. Introduction

Implementation of drive by wire strategies for the replacement of the conventional cable link between the throttle pedal and the throttle body has been the focus of development for many major automotive manufacturers. By fitting a stepper or permanent magnet servo motor [12] to the throttle body, and a throttle pedal with proportional voltage to position output, a "drive-by-wire" system can be implemented with a simple linear amplifier. If a microprocessor is added to the system, then control algorithms can be added to the operation of the throttle [13]. Controllers have been designed [11], which allow fast and accurate tracking of pedal demand, and have been shown to possess robust operating characteristics. An engine torque controller is designed and implemented in this paper to shape the vehicle response to the first torsional mode of the driveline. The initial requirement is to damp the acceleration oscillations generated by throttle step-demands. This dynamic mapping is constrained by the requirement to maintain where possible the vehicle acceleration response available to the throttle. Control analysis and design for this automotive system is complicated by a number of factors. There are a number of nonlinearities present, such as backlash in the gearbox, a tyre force which varies nonlinearly with road speed, and nonlinear clutch elements. Also, there is a process lag between throttle actuation and torque production and a nonlinear engine torque speed mapping [5]. Experimental data is available from a test car which was fitted with a data acquisition system including three axis accelerometers. A vehicle was loaned for the purpose of analysis, design and testing. This facilitated the development and verification of an accurate Matlab/Simulink dynamic model of the vehicle, driveline and engine. Although accelerometers were available for experimental verification, in the first instance it was desired that only signals and measurements available on a standard unmodified car be used in the control system. A systematic excitation of the driveline was made experimentally on the vehicle model by performing step demands in all gears at discrete points throughout the effective engine speed range of the vehicle. The generated data (road speed, acceleration, engine speed etc.) was then modelled using the "Response Surface Methodology" (RSM) [7]. The method allows the exploration and optimisation of response surfaces, where the response variable of interest (vehicle acceleration) is related to a set of predictor variables (road speed, selected gear). In the development of a model and control system constrained by computational considerations, and the requirement of rapid prototyping, the RSM allows a low order approximation to be derived [14] by the method of least squares. The reduced order representation can then be employed in the controller design. Application of the RSM analysis to the vehicle response data allows a system model to be developed which lends itself to the design of a scheduled controller structure which is shown to control the first torsional mode of the driveline.

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II. Metamodelling of the Simulink model

In the analysis of the acceleration response of the vehicle, there are three variables of interest, namely vehicle loading, road speed and selected gear ratio. Vehicle loading was assumed to be a fixed standard two person loading of $160kg$. In order to reduce the overall number of data sets required to construct the response surface of the system, a factorial approach to designed experiments was adopted [4]. The combinations of factorial experiments at $5ms^{-1}$ increments in each gear requires 25 experiments to be performed. Each proposed factorial combination was assigned a serial number and performed in random order via output from a random number generator. Each experiment consists of coasting the vehicle model in the appropriate gear at the appropriate roadspeed, and performing a 100% tip-in with the accelerator pedal. The simulated asphalt test road was assumed to be dry, with overcast sky and ambient temperature of 60^oF. The effect of the energy storage components in the driveline can be clearly seen at $10ms^{-1}$ in second gear in Figure 1. Examination of the vehicle response to tip-in reveals a system which can be approximated as a delay and second order dynamic response with an overshoot and settling time which varies with road speed and selected gear. This approximation to describe the entire vehicle response can be formulated by application of

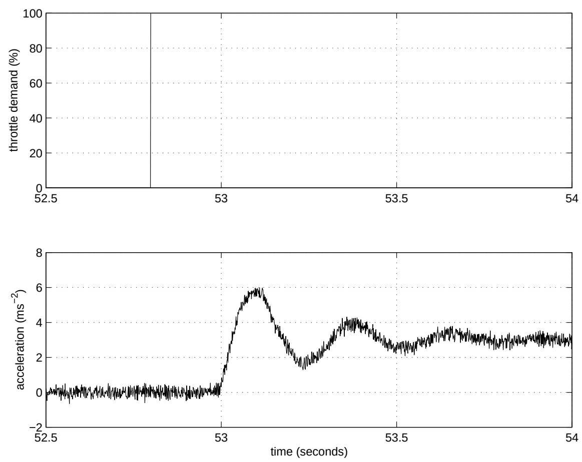

Fig. 1. Vehicle filtered experimental acceleration step response in second gear at $10ms^{-1}$.

RSM to the experimental data. The data which has been gathered can be synthesised into a response map for the vehicle in order to design an oscillation control system. For the definition of driveability under consideration here, the vehicle response can be characterised as the damping ratio of the second order approximation map with variables road speed and selected gear. The individual responses may be expressed in terms of overshoot and settling time. The transfer function describing the open loop system may be described as

$$\frac{C(s)}{R(s)} = k\frac{1}{as^2 + bs + c} \tag{1}$$

from which the damping ratio of the system can be calculated as the ratio of the actual damping b to the critical damping $b_c = 2\sqrt{ac}$ [8]. Thus the damping ratio ζ can be calculated from $\zeta = \frac{b}{b_c}$. The roots of the characteristic equation 1 are $s_1, s_2 = -b_c \pm jc\sqrt{1 - b_c^2}$. This forms a complex conjugate pair from which the damping ratio and natural frequency can be computed. The natural units ξ_1 and ξ_2 of the experimental data (road speed and selected gear) are first transformed into the corresponding normalised coded variables x_1 and x_2, such that

$$x_{i1} = \frac{\xi_{i1} - \left[max\left(\xi_{i1}\right) + min\left(\xi_{i1}\right)\right]/2}{\left[max\left(\xi_{i1}\right) - min\left(\xi_{i1}\right)\right]/2} \tag{2}$$

and

$$x_{i2} = \frac{\xi_{i2} - \left[max\left(\xi_{i2}\right) + min\left(\xi_{i2}\right)\right]/2}{\left[max\left(\xi_{i2}\right) - min\left(\xi_{i2}\right)\right]/2} \tag{3}$$

The model can be expressed in matrix form as [7]

$$\mathbf{y} = \mathbf{X}\beta + \epsilon \tag{4}$$

where

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1k} \\ 1 & x_{21} & x_{22} & \dots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nk} \end{bmatrix},$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_n \end{bmatrix}. \tag{5}$$

It is now necessary to find a vector of least squares estimators $\mathbf{b}$ which minimises the expression

$$L = \sum_{i=1}^{n} \epsilon_i^2 = \epsilon^{'}\epsilon = (\mathbf{y} - \mathbf{X}\beta)^{'}(\mathbf{y} - \mathbf{X}\beta) \tag{6}$$

and yields the least squares estimator of β which is

$$\mathbf{b} = \left(\mathbf{X}^{'}\mathbf{X}\right)^{-1}\mathbf{X}^{'}\mathbf{y} \tag{7}$$

and finally, the fitted regression model is

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b}, \quad \mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \tag{8}$$

where $\mathbf{e}$ is the vector of residual errors of the model.

The second order model to be fitted to the data is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \epsilon \tag{9}$$

Utilising equations (5-8), we obtain the coefficient matrix

$$b = \begin{bmatrix} 0.4079 \\ -0.804 \\ 0.3809 \\ 0.0519 \\ -0.0429 \\ -0.0121 \end{bmatrix} \tag{10}$$

therefore the response surface in terms of the coded variables is obtained.

$$\begin{aligned} \hat{y} &= 0.4079 - 0.804x_1 + 0.3809x_2 + 0.0519x_1^2 \\ &- 0.0429x_2^2 - 0.0121x_1x_2 \end{aligned} \tag{11}$$

Comparing the computed response surface against a second experimental testdata set gave an average residual error of 1.65%. The design of the prototype driveability compensator will be considered in the next section.

III. Rapid Prototype Design

A response surface has been obtained which describes accurately the vehicle's damping ratio map. As an initial design target, a damping ratio of 0.7 across the entire operating map would be a desirable response. The RSM analysis allows a simple open loop feedforward controller to be immediately designed and implemented to allow a fast appraisal of the actuator potential. The system response surface extracted from the experimental data is a representation of the complex conjugate pole pairs of the approximation (Figure 2) in terms of the system's varying damping ratio. The approach will be to effect a pole-zero cancellation of these complex conjugate poles to give

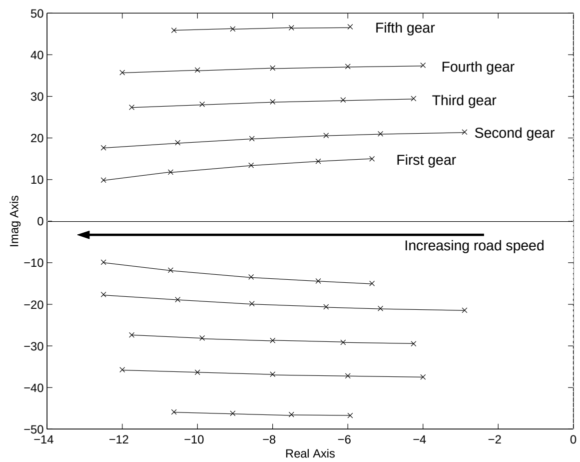

Fig. 2. Vehicle complex conjugate pole map.

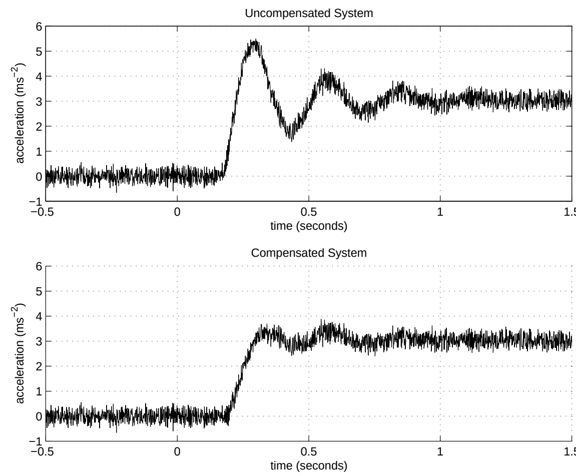

Fig. 3. Vehicle compensated and uncompensated filtered experimental step response.

a satisfactory response. This method does rely on accurate knowledge of the position of the uncompensated poles which has been ascertained experimentally for the purpose of this development. The parameters of the feedforward controller are derived from the response surfaces and are a function of selected gear and roadspeed. The feedforward compensator takes the form $\frac{as^2+bs+c}{as^2+ds+c}$ where the coefficient b is calculated from the damping ratio response surface, and performs pole-zero cancellation. Coefficient d produces the desired pole placement and forms the required damping ratio.

The control scheme was implemented on the microcontroller in assembly language, to ensure the fastest execution time. The demand from the throttle pedal and roadspeed were read in via A/D ports, and the selected gear read in via the digital I/O. Output from the controller was sent to a power amplifier via the PWM port. With the controller in place, the experimental set was on the vehicle to confirm the designed performance

IV. Experimental results - rapid prototyping

The goal of rapid prototyping was achieved in a matter of days between initial model approximation and final implementation testing. The original set of experiments were repeated on the vehicle with the electronic throttle both compensated and uncompensated. A comparison at the $10ms^{-1}$ in second gear step response is shown in Figure 3. The time axis in both traces was zeroed at the initiation of the step demand for the purpose of clarity. The marked improvement in vehicle oscillation obvious in figure **??** was repeated throughout the operating map of the vehicle. The smooth acceleration increase is in marked contrast to the results achieved with (for example) polynomial pole placement techniques [10] in which oscillation is present in the rising acceleration trajectory. The compensated vehicle responses over the operating region of the vehicle were found to have a mean damping ratio of 0.68, with a maximum residual of 0.07. The tip-in driveability of the vehicle was found to be subjectively very improved, in addition to the experimental evidence of the vehicle compensated step response. Although a small amount of acceleration rate is sacrificed to achieve the supression of oscillations subjectively the vehicles performance was not felt to be affected. The controller did endow the vehicle with a “turbine like” (a smooth positive rise in acceleration without any associated oscillations) feel.

V. Metamodelling for closed loop pole-placement design

In order to design a closed loop controller to achieve robustness against system parameter variations, a pole-placement approach was adopted. The objective of the pole-placement design method is to design a closed loop system with specified poles and thus the required dynamic response. The measured variable for feedback considered here is provided by a longitudinal accelerometer, should the derivation of acceleration from the velocity signal prove inaccurate or too noisy. The resulting characteristic equation will determine the features of the system, such as rise time, overshoot and settling time. The system model and its linear controller can be expressed respectively as

$$A(s)y(s) = B(s)u(s) \tag{12}$$

$$S(s)u(s) = T(s)u_c(s) - R(s)y(s) \tag{13}$$

where $A(s)$ and $B(s)$ are polynomials in the Laplace domain and $u(s)$ is the control variable. $S(s)$, $R(s)$ and $T(s)$ are the error, feedback and feedforward controller polynomials in the complex domain. The controller has input $u_c(s)$, which is the command signal and $y(s)$, is the measured output of the plant. Three constraints are associated with the model: the degree of $B(s)$ is less than the degree of $A(s)$, there are no common factors between polynomials $A(s)$ and $B(s)$, and $A(s)$is a monic polynomial. From equations 12 and 13, the characteristic equation of the closed loop system will be

$$F(s) = A(s)S(s) + B(s)R(s) \tag{14}$$

The objective of the pole placement design is to find polynomials $S(s)$ and $R(s)$ that satisfy equation 14 for specified $A(s)$, $B(s)$ and $F(s)$. Equation 14 is known as the *Diophantine equation* and can be solved if the polynomials do not have common factors and the system is proper. The Diophantine equation can be solved using a linear matrix. Two major questions arise with the use of the pole placement design, firstly what is the optimum location of the poles for the characteristic equation of the controller, and secondly how many poles must be placed? Resolving the issue may be a matter of trial and error if the system is complex and there is noise in the feedback signals or (as it is the case of the driveline) there are nonlinearities, such as delays and saturation curves. In this case, a multiobjective genetic algorithm was adopted to find the optimal pole placements. The design problem in this case is described by a five component objective function:

- minimise rise time
- minimise overshoot
- minimise settling time
- minimise steady-state error
- minimise delay

The experimentally elicited metamodel for the vehicle, in terms of damping factor for a second order fit to acceleration response is shown in figure 4. The polynomials $B(s)$

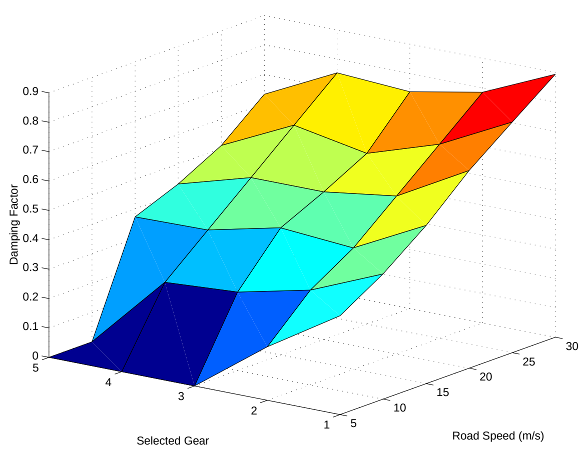

Fig. 4. Experimental vehicle damping ratio response surface.

and $A(s)$ can be obtained. For example, at $15mph$ in $2nd$ gear;

$$\frac{v(s)}{u(s)} = \frac{B(s)}{A(s)} = \frac{1615.7}{s^3 + 4.3s^2 + 521.8s} \tag{15}$$

As part of the model, a second order approximation will be added to equation 15 to model the delay relating to the manifold fill delay. The resulting polynomials were

$$\frac{B_1(s)}{A_1(s)} = \frac{B(s)P_{num}(s)}{A(s)P_{den}(s)} \tag{16}$$

where $A_1(s)$ and $B_1(s)$ are the new polynomials representing the system and $P_{num}(s)$ and $P_{den}(s)$ are the numerator and denominator respectively of the Pade approximation. The order difference in the process polynomials is $\alpha = n_a - n_b = 5 - 2 = 3$, and defining $\beta = 0$, the order of the controllers $S(s)$ and $R(s)$ and the closed loop characteristic equation $F(s)$ can be found as in equation 14. Then, $n_s = 4$, $n_r = 4$ and $n_c = 9$. This leads to the matter of determining the value of nine roots for the characteristic equation. The lower order approximations have been used to determine a tractable number of poles to place in the controller design. The original Matlab/Simulink model is computationally too slow for the iterative procedure inherent in evolutionary optimisation. Consequently the metamodel is adopted for its speed of execution. The pole locations will be the decision variables in the multi objective genetic algorithm (MOGA), since those are the unknowns in equation 14 and was used to calculate directly the values of the coefficients for the polynomials $R(s)$, $T(s)$ and $S(s)$. The gain of $T(s)$ was also be included as a decision variable. The coding of the variables was real, since that allows more natural data handling and is more efficient. The objectives set the goals to reach and ensure that every selected individual satisfies the specifications. The performance objectives have already been described, and relate to overshoot, settling time etc. The initial conditions (selected gear, road speed) in addition to reasonable real-life variations in the mechanical parameters (lash etc.) for the driveline nonlinear model were changed randomly for each individual in each generation, in such a way that the best controllers are the ones that could perform adequately under wide system parameter and condition variations. The variation in lash in particular replicates one of the fundamental characteristics of the ageing of the system. A further addition to this approach was the variation in road conditions and vehicle loading from one to four occupants. This approach was intended to achieve as far as possible, a robust controller. Finally, the minimisation of the control energy was included amongst the objectives in order to achieve a feasible, efficient controller. The bounds of the random variations were as follows:

- lash 0^o to 30^o at wheels
- road conditions from μ 0.4 to 1
- vehicle loading 1 to 4 occupants (standard occupant = $80kg$
- road gradient -10 to $+10\%$

This bounding set was later utilised to assess the robustness of the robust controller. The GA Toolbox for Matlab with the MOGA extension tools developed at the University of Sheffield [3] was utilised to perform the search procedure. The decision variables are in this case assigned to the controller pole placement positions.

VI. Metamodelling for evolutionary optimisation - results

A candidate controller of the form;

$$\begin{aligned} R(s) &= s(s - 20.1 + 12.1i)(s - 20.1 - 12.1i) \\ &\quad (s - 17.3) \\ S(s) &= (s - 167.8)(s - 41.3 + 103.2i) \\ &\quad (s - 41.3 - 103.2i)(s - 18.6) \\ T(s) &= (s - 40.6 + 3.3i)(s - 40.6 - 3.3i) \end{aligned}$$

$$(s - 37 + 1.71i)(s - 37 - 1.71i) \tag{17}$$

was selected from the family of potential solutions on the basis of its minimisation of all the objectives stated in the objective function, and its overall driving "feel". The controller was simulated under varying initial conditions and mechanical parameters to verify its performance, and also assess its robustness. The predicted effects of ageing (for example on lash) were found in simulation of the closed loop system to produce acceleration responses which were within the bounds of desired "driveability".Although experimental assessment of the controller in terms of varying lash was not possible, a number of step responses were taken under varying vehicle loading. A factorial study

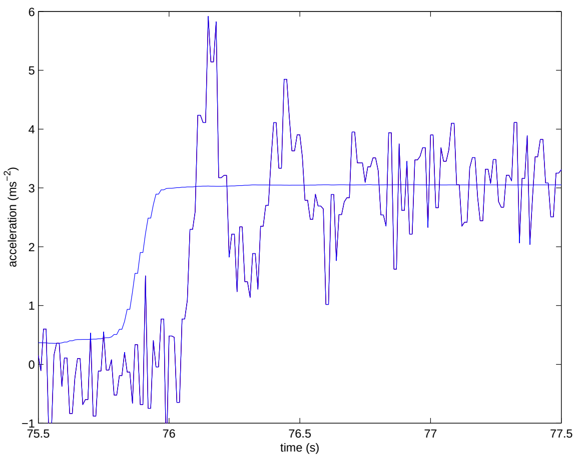

Fig. 5. Open loop unfiltered experimental step response, 1 passenger, 15mph, 2nd gear.

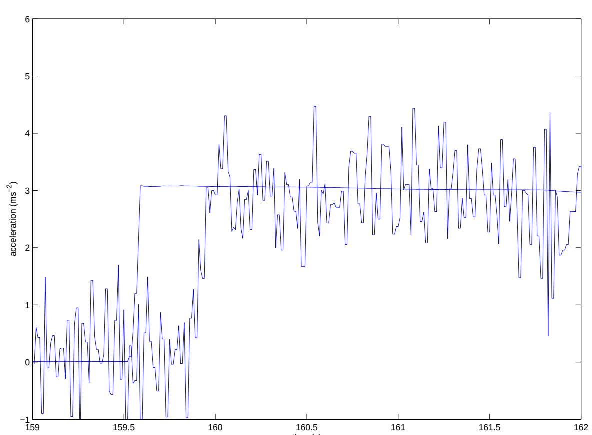

Fig. 6. Closed loop unfiltered experimental step response, 1 passenger, 15mph, 2nd gear.

was undertaken at a range of road speeds and selected gear ratios, with vehicle loading being chosen randomly. A typical open loop response is shown in figure 5 with the corresponding closed loop response being shown in figure 6 . The response is satisfactory in terms of driveability both in the step response and the delay time of the vehicle acceleration. The controller was found to be robust to changes in vehicle loading, and showed an excellent response across the vehicles entire operating range. Although it was not possible to vary the vehicle lash, simulation results predict that the controller is again robust for levels of lash up to 30^o at the wheels.

VII. Conclusions

This project had two initial objectives, firstly to examine the application of the Response Surface Methodology to rapid control design prototyping, and having successfully achieved this goal, the second objective was to use the RSM to examine the potential of electronic throttle control to shape the acceleration response of an experimental vehicle. The timescale from experimental data capture through RSM analysis to experimental verification and assessment of the electronic throttle's potential for vehicle acceleration shaping was under four days. At this point, a judgement upon the viability and potential of the project could easily be made with confidence. In this context, the use of metamodelling becomes extremely attractive. The technique has allowed an examination of the system control potential to be made at the start of the project, benefitting both the confidence of the industrial partners, and giving a realistic benchmark of potential performance. The controller derived by the RSM is immediately useable on the experimental vehicle, providing a demonstration facility at project inception. Some other benefits of this development tool are also significant. A controller is quickly available for verifying the mechanical and electrical components of the control system, giving a stable platform for subsequent controllers as and when they become available. The controller as designed via the RSM is simple and low order by nature, and thus can be installed on a very simple microcontroller. Finally, a quick and cheap assessment of a system's potential can be rapidly made in order to support project development proposals. The design of a useable implementation has been achieved on a time varying process with significant time delays and nonlinearities. A closed-loop controller was subsequently derived using the pole placement method. Multi objective evolutionary algorithms were applied to find the optimal location of the poles for the characteristic equation. Random initial conditions were applied to each generation to achieve robust solutions. The response of the selected controller shows a dramatic improvement over the open loop response, and does not require tuning depending on variations in the system parameters. The controller response also proves to have a better performance than the results obtained in the literature. The combination of the pole placement method with MOGA as a technique for driveline controller optimisation results in an efficient design procedure, where the lack of knowledge of the possible solutions does not necessarily affect the result of design process. Although an accelerometer was fitted to the vehicle for verification purposes, the implemented controller worked with the vehicle speed feedback signal which was readily available.

The effect of the closed loop controller upon the driver and passengers was perceived to be extremely beneficial. It was possible to repeat driven routes with the controller both engaged and disengaged. Although this method is extremely subjective when compared to rise-time/overshoot/settling time analysis, for the end user (a variety of drivers), the effect of the controller was found to give a distinct improvement to the driving experience.

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