DERIVING A HYBRID ALGORITHM TO SOLVE HEAT FLOW PROBLEMS

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Abstract: The Heat and Fluid Flow problems commonly occurring within the design of heat exchangers are difficult and time consuming to solve using traditional techniques. This paper describes an investigation into the suitability of the use of heuristic optimisation techniques, Genetic Algorithms, Tabu Search and Simulated Annealing, in the solution of complex heat flow problems. The derived Hybrib Algorithm was constructed by combining elements from all three approaches and the final algorithm acted to reinforce their strengths and minimise their weaknesses to produce an efficient and effective algorithm.
Keywords: Heuristic Methods, Hybrid Techniques, Heat Flow.

1 PROBLEM DEFINITION

The heat flow across a slab, ABCD see figure 1

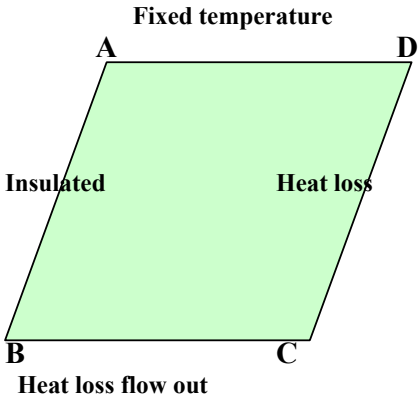

figure 1

is modelled by the partial differential equation:-

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

which together with the respective boundary conditions:-

AB $$\frac{\partial T}{\partial x} = 0$$

BC $$k\frac{\partial T}{\partial y} = -\phi_0$$

CD $$-k\frac{\partial T}{\partial x} = h\left(T - T_0\right)$$

DA $$T = T_0$$

enables the determination of the heat profile, steady state temperatures, across the slab.

2 SOLUTION METHODOLOGY

The solution to this problem was derived using elements from:-

Genetic Algorithms
Simulated Annealing
Tabu Search

The final form of the algorithm was constructed to take advantage of the strengths of each of these methods and to minimise the effect of their weaknesses.

2.1 The initial GA approach

The initial stage was concerned with an investigation into the use of Genetic Algorithmic techniques, thus the key stages in a Genetic Algorithm approach to solving this problem had to be defined, these were: -

The form of the GA strings
Here each string consisted of 24 real numbers, each representing the temperature at one of the grid points on the slab.

Selection for crossover
The strings were selected on a proportional basis, to their fitness, to be included in the population used to generate the next set of strings, using crossover.

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The crossover technique
In this problem a block cross over was employed, that is the information from a randomly chosen block was exchanged by the chosen pair of strings. This method was employed because it better represented the geometry within the problem.

Elitist approach
Following crossover one of the new strings was chosen randomly to be replaced by the existing best string.

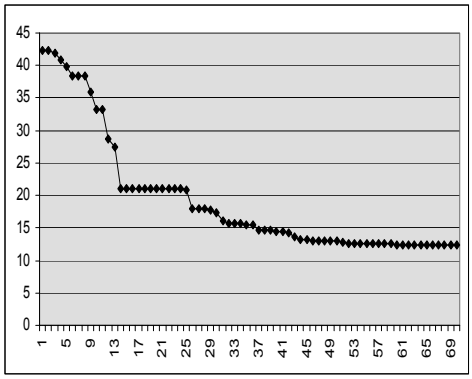
2.1.1 Implementation

These procedures were combined to give the basic approach this was transformed into an iterative approach through the addition of the routine:-

when the (GA) process has converged, a new set of strings close to the existing best solution are generated and the process repeated, thus leading to an Iterative Genetic Algorithmic (IGA) approach to this problem, note that the search space is reduced at each successive iteration.

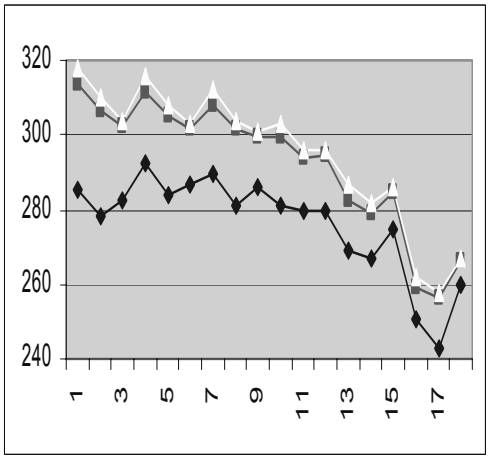
2.1.2 Results

These procedures were applied with varying sizes of initial population and to problems of varying difficulty, where difficulty is assumed to be related to the range of temperatures in the initial population.
Regardless of the initial population the Genetic Algorithm adopted an asymptotic behaviour, but tending towards a residual of10, the target was to obtain a residual of about 0.1. See graph 1.



Graph 1:Residual values

The Genetic Algorithm acted to produce solutions which were aligned with (parallel to) the temperatures at the optimal solution, but it was not able to move very close to the optimal solution, see Graph 2 where the "top set of points" is at the solution, the lower set of points is the converged solution, and the third set of points has a residual of less than 1.

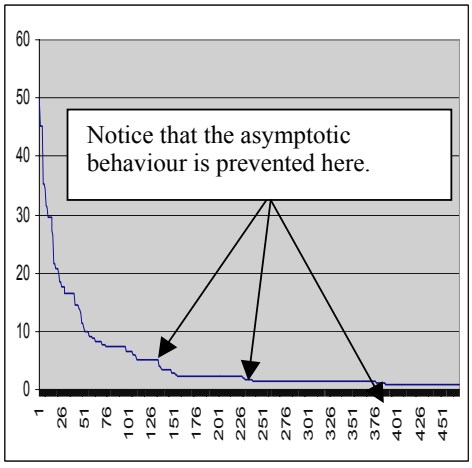


Graph 2: Temperatures across the slab

Thus it can be seen that although the Iterative Genetic Algorithm did not in itself lead to a satisfactory solution methodology it did act to "line up" the temperatures across the slab. Therefore this approach was adapted by incorporating elements from Simulated Annealing to attempt to overcome this convergence problem.

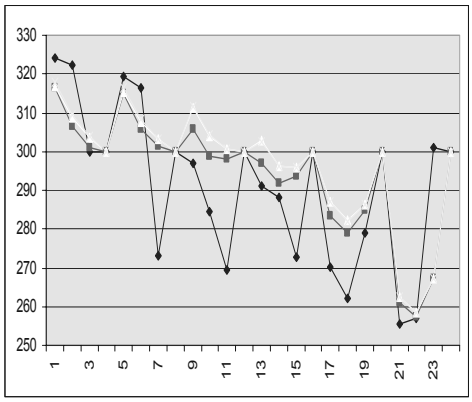
2.2 Refined Iterative Genetic Algorithm (RIGA)

Aspects of Simulated Annealing are introduced into the methodology at the stage when a new solution set is generated. In the IGA approach the search space is narrowed at each iteration, in the refined methodology, the search space is randomly widened. This has the effect of overcoming the asymptotic "early" convergence of the IGA approach, replacing Graph 1 with Graph 3.



Graph 3: Residual values using restart

This procedure has acted to overcome the original asymptotic "early convergence" enabling the process to produce a solution with a residual less than 0.1. See graph 4 where the optimal solution is indicated by the "triangular point markers".



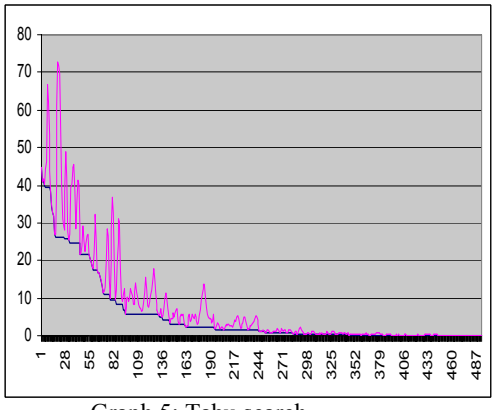
Graph 4: Temperatures across plate

2.3 The Hybrib Algorithm

The RIGA obtained a solution to the problem to the required accuracy however the implicit asymptotic behaviour of the GA acted to slow down the process. Therefore the next stage in the development of the hybrib algorithm investigated the incorporation of a Tabu Search.

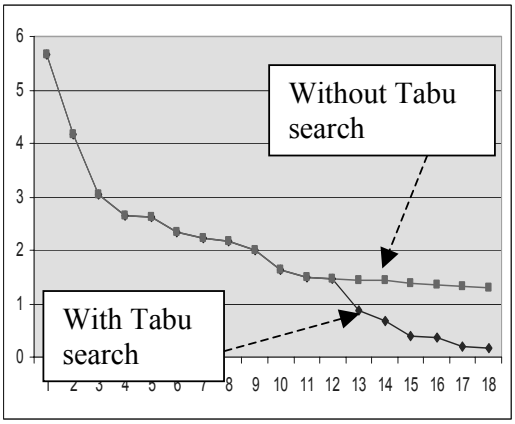
The RIGA algorithm was employed until the residual was reduced to a value between 0.5 and 1.5, (where the asymptotic behaviour has not yet started to dominate the process). At this point a varying step size Tabu Search is implemented.

Graph 5 shows the search space investigated by the Tabu Search procedure where the current point is shown together with the current best point.



Graph 5: Tabu search

The Tabu search acted to improve the search for the optimal solution, see graph 6. This shows how the Tabu Search prevents the asymptotic behaviour of the RIGA algorithm, leading to the Hybrid Algorithm (MA).



Graph 6: Tabu search effect on Residual

3 COMPARISONS BETWEEN THE APPROACHES:

To enable comparisons between the two approaches RIGA, and the Hybrid Algorithm (RIGA incorporating Tabu Search), the run times have been normalised so that the Tabu search requires 1 time unit.

Six problems were solved using both approaches. The final residual obtained and the time to solve the problems were averaged and these results are summarised in Table 1.

Results	RIGA	MA
Average final residual	0.06	0.06
Time MA = 0.57 * Time RIGA		

Table 1: Comparing Methods

Thus this algorithm works efficiently to obtain the solution to the heat flow problem. The algorithm acts to combine the strengths of each of the constituent techniques and minimises their weaknesses.

4 CONCLUSIONS

This work has demonstrated the fact that heat flow problems can be solved using modern optimisation techniques. However it has also shown that the solution algorithm has to be derived around the needs of the problem under consideration. Here a Genetic Algorithm, alone, could not produce a satisfactory solution (not enough accuracy) but it could indicate the answer, see graphs 2 and 4 where the GA

found solutions which were parallel with the optimal solution. Therefore once this state has occurred, which relates to the asymptotic behaviour of the GA, an alternative methodology, a search methodology can be employed to get to the solution, here a Tabu search was used. Thus the efficient solution methodology, a Hybrid Algorithm, was derived by combining elements from a set of methods. This methodology is now being extended into the study of more complex heat flow problems.

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