FUZZY MODELLING APPLIED TO JOBSHOP SCHEDULING

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Abstract: Fuzzy set theory allows the complexity of real-life issues to be included within the confines and rigours of the mathematical model. The authors have applied fuzzy methodology to the scheduling of jobs, the objective being the determination of an optimal sequence for dynamic job arrivals such that potentially conflicting priorities are satisfied. This paper concentrates on the theory on which the models are based, demonstrating an application by referring to a static problem.
Keywords: Scheduling, Fuzzy Logic, Jobshop.

1. INTRODUCTION

1.1 Scheduling

A scheduling problem can be considered to be an exercise in finding an appropriate timetable for the processing of jobs, by machines, such that some performance measure achieves its optimal value. Within this definition, it can be seen that there are two aspects to be considered concurrently, the satisfaction of constraints (e.g. availability of resources) and the optimisation of objectives (e.g. flow-times).

In general, such problems are known to be NP hard and probably as a consequence of this, scheduling has been an active area of research for many years. However, Pinedo [1995] notes, real-world scheduling problems are usually very different from the mathematical models studied by researchers in academia. Panwalker & Iskander [1977] also reported on the discrepancy between performance measures used by researchers and those preferred in industry. Actual firms place a higher priority on meeting due-dates than on typical research objectives such as minimising flow-time. (Gee & Smith [1993])

Woolsey [1982] also warns of the dangers inherent in failing to take a holistic view of production scheduling.

Pinedo lists a number of important requirements of real-manufacturing that are not normally met by OR models. An example of this is the existence of multiple objectives, i.e. there is not a single objective but multiple objectives to be optimised.

For example, given a jobshop with random job arrivals, which processes all jobs on a single machine, (the scheduler may need to consider the following goal:

Satisfy all due-dates, however certain jobs are for particularly important customers and it is a major priority to ensure that these jobs are completed on time.

1.2 The System

The authors have applied fuzzy methodology to the scheduling of jobs, the objective being the determination of an optimal sequence for dynamic job arrivals such that potentially conflicting priorities are satisfied. This paper concentrates on the theory on which the models are based, demonstrating an application by referring to a static problem. The main focus is on the study of a jobshop processing all jobs on a single machine. The difficulty in scheduling these jobs arises as a consequence of the existence of a multi-criteria objective, i.e. to meet all due-dates, whilst ensuring the satisfaction of the most significant customers.

The relevance of a fuzzy logic approach can be justified in the desire to optimise multiple objectives and so achieve a closer resemblance to the real-world. (Zadeh [1973], in his paper *Outline of a New Approach to the Analysis of Complex Systems & Decision Processes,* proposed that conventional quantitative techniques of system analysis are unsuited to dealing with humanistic systems.)

2. FUZZY SCHEDULING

2.1 Fuzzy modelling

Fuzzification

The first stage in producing a model, is to identify those linguistic variables to be included. It was decided that *due-date* and *customer priority* were the most significant factors, with *processing time* being of lesser importance.

Due-date

The actual allocation of due-dates was deemed to be outside the control of the scheduler. This is frequently the case in reality, due-dates frequently being given to customers by sales personnel without reference to the production

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staff. Consequently in line with this practice, every job was allocated a due-date of 28 days from its arrival time.

The relevance of due-date to the scheduler was assumed to be in terms of 'close' and 'distant'.

Hence given the universe $U = [-\infty, 28] \in \mathbf{Z}$, and the fuzzy sets C = CLOSE and D = DISTANT, the membership functions can be defined as below.

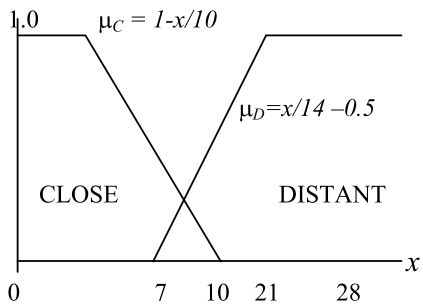

Fig. 1 Membership of CLOSE and DISTANT

Membership of CLOSE

$\mu_C(x) = 1.0$	$x \le 0$
$\mu_C(x) = 1.0 - x/10$	$0 < x < 10$
$\mu_C(x) = 0$	$x \ge 10$

Membership of DISTANT

$\mu_D(x) = 0$	$x \le 7$
$\mu_D(x) = x/14 - 0.5$	$7 < x < 21$
$\mu_D(x) = 1.0$	$21 \le x \le 28$

The selection of a 'trapezoidal' form of membership function for 'close' is based on the assumption that the criticality of the closeness of an impending due date increases linearly with time up to the point at which the job becomes 'late'. The 'distant' function represents a wish to avoid too early completion causing stock holding problems. The linear representation of increasing (and decreasing) closeness (and distance) has been selected, not only as a practical modelling assumption, but also as an appropriate one, in the absence of any established evidence of a need for a more complex (e.g. quadratic) form.

Customer Priority

The universe of discourse was deemed to be the set of 'customer ratings', {Bad, Low, Medium, High, Very Important}, with membership of the fuzzy set CP = CUSTOMER PRIORITY taking the form:

μ_{CP}	0.0	0.2	0.5	0.75	1.0
Cp	bad	low	Med	High	Very Important

μ_{CP} (Bad) = 0.0
μ_{CP} (Low) = 0.20
μ_{CP} (Medium) = 0.50
μ_{CP} (High) = 0.75
μ_{CP} (Very Important) = 1.0

Processing Time

It is assumed that at the time of scheduling the *exact* processing times are unknown, (Hestermann & Wolber[1997]). However the scheduler can estimate a processing time as 'short', 'medium' or 'long'. Thus the following membership functions are defined for the fuzzy sets SHORT, MEDIUM and LONG.

$U = [0, 14] \in \mathbf{Z}$

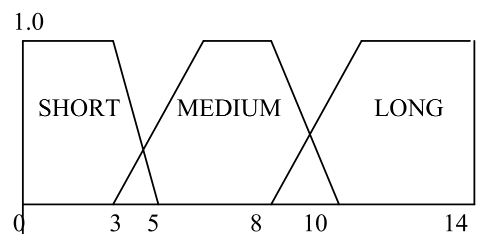

Fig. 2 Membership of SHORT, MEDIUM and LONG

Rule Evaluation

The fuzzy inputs of CLOSE, DISTANT and CUSTOMER PRIORITY are combined to produce an output which is a sequence priority. (Table 1)

Rule Matrix

Due-Date / **Customer Priority**	**Close**	**Distant**
Bad (B)	Reject	Reject
Low (L)	Sequence quite high	Sequence very low
Medium (M)	Sequence high	Sequence low
High (H)	Sequence very high	Sequence quite low
Very important (VI)	Sequence extremely high	Sequence medium

Table 1 Summary of Sequencing Priorities

For example:

IF *customer priority* is Bad AND *due-date* is Close THEN Reject.

IF *customer priority* is Low AND *due-date* is Close THEN Sequence quite high.

IF *customer priority* is High AND *due-date* is Distant THEN Sequence quite low.

Sequencing Priority

Sequence -
Extremely high EH
Very high VH
High H
Quite high QH
Medium M
Quite low QL
Low L
Very low VL
Reject R

Table2 Ordering of sequence priorities

If more than one job has the same priority at the head of the sequence, then a job with 'shortest' processing time will be selected for processing.

The general model

Composition or relational product:

Suppose $T = S \circ R$,

where $R \in F(X \times Z), S \in F(Z \times Y)$.

$\forall (x, y) \in X \times Y$:

$\mu_T(x, y) = \sup_{z \in \mu_2} \min \{\mu_R(x, z), \mu_S(z, y)\}$ [1]

Union

$X = A \cup B \Leftrightarrow \forall x \in U$

$[\mu_X(x) = \mu_A(x) \vee \mu_B(x)]$

$= \forall x \in U \; [\mu_X(x) = \max\{\mu_A(x), \mu_B(x)]$ [2]

The fuzzy relation SP representing the sequencing priorities, is derived from an application of equation [1]

$\mu_{SP}(c, d) = \underset{cp \in CUSTPRI}{\textbf{supmin}} \{\mu_R(c, cp), \mu_s(cp, d)\}$ [3]

where R $\in$ *F* (CLOSE $\times$ CUST-PRI)

S $\in$ *F* (CUST-PRI $\times$ DISTANT)

and $SP = S \circ R$.

2.2 Application

A hand-worked example will illustrate how the rule base enables a job to improve its sequencing priority as the due-date gets closer. Note, however that a job for a Bad customer will be rejected and not included in the sequencing schedule. The following example will illustrate the mechanics of the fuzzy algorithm. There are six jobs waiting to be processed, one of which is for a customer considered to be of 'medium' importance and two for 'very important' customers.

The example has been deliberately chosen to create problems for the scheduler in the light of conflicting priorities, i.e. of fulfilling all promised due-dates whilst ensuring the satisfaction of the most significant customers.

The due-dates range from 0 days for Job 1 (medium) to 28 days for Job 4 (very important).

The fuzzy values for 'customer priority', 'close' and 'distant' have been derived according to the definitions given in §2.1 .

The sequencing priority is then determined by applying equation [3] in the form:

$\mu_{SP}(c, d) = \min\{\mu_c, \mu_{cp}\} \vee \min\{\mu_{cp}, \mu_d\}$,

according to the rule matrix in Table 1.

Job	1	2	3	4	5	6
Due-date	0	10	6	28	26	14
Process time	5	1	8	6	2	4
Cust. Priority	M	L	V. I	V. I	H	L
Fuzzy cust-pri	0.5	0.2	1.0	1.0	0.75	0.2
Fuzzy dd-close	1.0	0.0	0.4	0.0	0.0	0.0
Fuzzy dd-dist	0.0	0.21	0.0	1.0	1.0	0.5
Max-min	clse	dist	clse	dist	dist	dist
Seqnce:	H	VL	E H	M	Q L	VL

Table 3: Test example – Six jobs waiting to be processed.

Step 1

Consider Job 1:

Comparing the fuzzy value of 'customer priority' with 'close' and 'distant' –

$\min\{\mu_c, \mu_{cp}) \vee \min\{\mu_{cp}, \mu_d\}$

$\mu_{cp} = 0.5$ (customer priority is Medium)

$\mu_c = 1.0$ (membership of 'close')

$\mu_d = 0.0$ (membership of 'distant')

$(\min\{1.0, 0.5\} = 0.5) \vee (\min\{0.5, 0.0\} = 0.0)$

$\max\{0.5, 0.0\} = 0.5$ 'close'

(Application of equation [2])

Thus: Medium and close =>

Sequence high; μ_{SP} (according to Table 1)

Job 2: $\min\{0.0, 0.2\} = 0.0$

$\min\{0.2, 0.21\} = 0.2$

$\max\{0.0, 0.2\} = 0.2$ 'distant'

Thus: Low and Distant =>

Sequence very low

Job 3: $\min\{0.4, 1.0\} = 0.4$

$\min\{1.0, 0.0\} = 0.0$

max {0.4, 0.0} = 0.4 'close'
Thus:Very Important and Close=>
Sequence extremely high

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 1.0} = 1.0
max {0.0, 1.0} = 1.0 'distant'
Thus: Very Important and Distant =>
Sequence medium

Job 5: min {0.0, 0.75} = 0.0
min {0.75, 1.0} = 0.75
max {0.0, 0.75} = 0.75 'distant'
Thus: High and Distant =>
Sequence quite low

Job 6: min {0.0, 0.2} = 0.0
min {0.2, 0.5} = 0.2
max {0.0, 0.2} = 0.2 'distant'
Thus: Low and Distant =>
Sequence very low

The sequencing priority is given by:
< 3, 1, 4, 5, 2, 6 >Job 3 (the head of the sequence) is processed – duration 8 days..

Step 2

Step 2 will repeat all the tasks in Step 1, for the remaining five jobs.
All the due-dates are adjusted:
due-date(new) = due-date(old) – process time(job 3)

Job	1	2	4	5	6
Due-date	-8	2	20	18	6
Process time	5	1	6	2	4
Cust. Priority	M	L	V. I	H	L
Fuzzy cust-pri	0.5	0.2	1.0	0.75	0.2
Fuzzy dd-close	1.0	0.8	0.0	0.0	0.4
Fuzzy dd-dist	0.0	0.0	0.93	0.79	0.0
Max-min	close	close	dist	dist	close
Sequence:	H	QH	M	QL	QL

Table 4: Test example – Five jobs in queue.

Job 1: min {1.0, 0.5} = 0.5
min {0.5, 0.0} = 0.0
max {0.5, 0.0} = 0.3 'close'
Thus: Medium and Close =>
Sequence high

Job 2: min {0.8, 0.2} = 0.2
min {0.2, 0.0} = 0.0
max {0.2, 0.0} = 0.2 'close'

Thus: Low and Close =>
Sequence quite high

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 0.93} = 0.93
max {0.0, 0.93} = 0.93 'distant'
Thus: Very Important and Distant =>
Sequence medium

Job 5: min {0.0, 0.75} = 0.0
min {0.75, 0.79} = 0.75
max {0.0, 0.75} = 0.75 'distant'
Thus: High and Distant =>
Sequence quite low

Job 6: min {0.4, 0.2} = 0.2
min {0.2, 0.0} = 0.0
max {0.2, 0.0} = 0.2 'close'
Thus: Low and Close =>
Sequence quite high

The current sequencing priority is given by:
< 1, 2, 6, 4, 5 > thus Job 1 is processed – duration 5 days.

Step 3:

Job	2	4	5	6
Due-date	-3	15	13	1
Process time	1	6	2	4
Cust. Priority	Low	V. Imp	High	Low
Fuzzy cust-pri	0.2	1.0	0.75	0.2
Fuzzy dd-close	1.0	0.0	0.0	0.9
Fuzzy dd-dist	0.0	0.57	0.43	0.0
Max-min	close	Distant	distant	Close
Sequence:	Quite high	Medium	quite low	quite high

Table 5: Test example – Four jobs in queue.

Job 2: min {1.0, 0.2} = 0.2
min {0.2, 0.0} = 0.0
max {0.2, 0.0} = 0.2 'close'
Thus: Low and Close =>
Sequence quite high

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 0.57} = 0.57
max {0.0, 0.57} = 0.57 'distant'
Thus: Very Important and Distant =>
Sequence medium

Job 5: min {0.0, 0.75} = 0.0
min {0.75, 0.43} = 0.43
max {0.0, 0.43} = 0.43 'distant'
Thus: High and Distant =>
Sequence quite low

Job 6: min {0.9, 0.2} = 0.2
min {0.2, 0.0} = 0.0
max {0.2, 0.0} = 0.2 'close'
Thus: Low and Close =>
Sequence quite high

The sequencing priority for the current jobs is now: < <u>2, 6,</u> 4, 5 >

Jobs 2 and 6 have the same sequencing priority, so the algorithm considers the *estimated* process time.

Job 2 would be classified as 'short'

($\mu_{SHORT}(j_2) = 1.0$),

Job 4 has a probability of 0.5 of being estimated as 'short',

($\mu_{SHORT}(j_4) = 0.5$, $\mu_{MED}(j_4) = 0.5$).

Job 2 would be chosen for processing – duration 1 day.

<u>Step 4:</u>

Job	4	5	6
Due-date	14	12	0
Process time	6	2	4
Cust. Priority	V. Imp	High	Low
Fuzzy cust-pri	1.0	0.75	0.2
Fuzzy dd-close	0.0	0.0	1.0
Fuzzy dd-dist	0.5	0.36	0.0
max-min	Distant	distant	close
Sequence:	Medium	quite low	quite high

Table 6: Test example – Three jobs in queue.

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 0.5} = 0.5
max {0.0, 0.5} = 0.5 'distant'

Thus: Very Important and Distant => Sequence <u>medium</u>

Job 5: min {0.0, 0.75} = 0.0
min {0.75, 0.36} = 0.36
max {0.0, 0.36} = 0.36 'distant'

Thus: High and Distant =>
Sequence <u>quite low</u>

Job 6: min {1.0, 0.2} = 0.2
min {0.2, 0.0} = 0.0
max {0.2, 0.0} = 0.2 'close'

Thus: Low and Close =>
Sequence <u>quite high</u>

The sequencing priority is given by:
< 6, 4, 5 >
so Job 6 is processed, - duration 4 days.

<u>Step 5: (see table 7)</u>

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 0.21} = 0.21
max {0.0, 0.21} = 0.21 'distant'

Thus: Very Important and Distant =>
Sequence <u>medium</u>

Job 5: min {0.2, 0.75} = 0.2
min {0.75, 0.07} = 0.07
max {0.2, 0.07} = 0.2 'close'

Thus: High and Close =>
Sequence <u>very high</u>

Job	4	5
Due-date	10	8
Process time	6	2
Cust. Priority	V. Imp	High
Fuzzy cust-pri	1.0	0.75
Fuzzy dd-close	0.0	0.2
Fuzzy dd-dist	0.21	0.07
max-min	Distant	close
Sequence:	Medium	very high

Table 7: Test example – Two jobs in queue.

Job 4: min {0.0, 1.0} = 0.0
min {1.0, 0.21} = 0.21
max {0.0, 0.21} = 0.21 'distant'

Thus: Very Important and Distant =>
Sequence <u>medium</u>

Job 5: min {0.2, 0.75} = 0.2
min {0.75, 0.07} = 0.07
max {0.2, 0.07} = 0.2 'close'

Thus: High and Close =>
Sequence <u>very high</u>

This gives the final sequencing priority:
< 5, 4 >
Job 5 is processed, - duration 2 days.

Thus the complete schedule is defined as:

< 3, 1, 2, 6, 5, 4 >

and is summarised in the following table:

Job	3	1	2	6	5	4
Cust. Priority	V. I	M	L	L	H	V. I
Due-date	6	0	10	14	26	28
Process time	8	5	1	4	6	2
Start time	0	8	13	14	18	20
Completion time	8	13	14	18	20	26
Lateness	2	13	4	4	-6	-2

Table 8: Test example – Final schedule.

The job completion times for the most important customers (V. Imp and High) are satisfactory.

The main cause for concern, at first glance, is the 13 day lateness attributed to the 'medium' rated customer. However closer scrutiny reveals that this was unavoidable, the algorithm correctly gave priority to the 'very important' job.

The dynamic process can be summarised by considering the sequence priority each time the machine becomes available:

<3,1,4,5,2,6>	Job 3 processed
<1,2,6,4,5>	Job 1 processed
<2,6,4,5>	Job 2 processed
<6,4,5>	Job 6 processed
<5,4>	Job 5 processed

2.3 Further Development

A model for fuzzy decision making which considers conflicting scheduling priorities, has been described. Further enhancement/refinement could be incorporated. For example, additional fuzzy variables associated with 'earliness' and 'lateness' could be considered for inclusion in the algorithm, in order to allow consideration of stock-holding costs to be included in the model.

An increase in demand naturally leads on to consideration of the use of two or more machines. A second model for a multi-machine problem considered the availability of two machines with the following properties:

Machine A:
Cheap to run, but incurs longer process times.

Machine B:
Expensive to run but incurs shorter process times.

At times of light or normal demand, jobs would be processed on Machine A, the alternative action,
process job on Machine B, could be triggered as heavier demand causes queue build-up.

A typical inference rule would be:

IF Number of jobs in queue is *heavy*
OR Number of jobs with sequence priority ≥ medium is *normal*
THEN Action

The antecedent of the rule is represented by an application of *union*, equation [2].

The consequent of the rule would be:
Action
Process head of sequence on Machine B

The universe of discourse is N.
Membership of the fuzzy subsets *light, normal* and *heavy* is defined according to Figure 3:

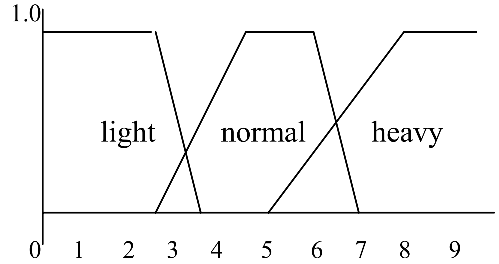

Fig 3 Fuzzy sets associated with queue state

Examples

1. Suppose there are 3 jobs currently in the queue, all have customer priority rating of 'medium' or above, then no action.

2. Suppose there are 5 jobs in the queue, 4 of which have customer importance rated as 'medium' or higher, then action.

3. Suppose there are 7 jobs in the queue, only 3 jobs with customer importance rating of 'medium' or higher, then action.

3. CONCLUSIONS

Fuzzy set theory allows the complexity of real-life issues to be included within the confines and rigours of the mathematical model. In this paper, a theoretical model has been presented which demonstrates how fuzzy decision making can support the dynamic scheduling process, enabling the conflicting priorities of multi-objectives to be managed effectively in polynomial time.

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