LEACHABLE GEOMETRY

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KEYWORDS
Computational Geometry, Salt Leaching, Dynamic Geometry, Hydro-Geochemistry

ABSTRACT

Most geoscientifical problems have a geometrical component respective representation. Depending on the requirements of the problem which need to be modeled, an optimal geometrical model needs to be chosen. Then, several properties have to be taken into account: differentiability, regularity, modifiability, controllability, extendibility and the possibility to attach a physical model. A geometrical model which supports all of these properties is yet to be invented. While one model for instance might support differential geometry very well, it may lack the ability to interpolate complex geometry, which another model does, while lacking in turn differentiability. This paper summarizes the properties of the most common geometrical models for solid geometry in 3D in the context of the modeling of the hydro-geochemical process "salt leaching in flooded potassium mines". Hereby emphasis is placed on model topology and model dynamics. Additionally, consequences for geometrical modeling due to the fundamental differences between geochemical and physical based modeling are pointed out.

INTRODUCTION

The subject of this paper is related to the interdisciplinary (comprising Geo-Technology and Computer Science) project cluster *Development and Application of ICT-based Methods for the Impact Analysis, Prognosis and Control of anthropogenic influenced Processes in Geosystems* supported by the DFG (German Research Foundation).

Geoscientific background for the described investigations is an already for decades lasting underground salt leaching process in the area of Stassfurt/Germany. There, potassium bearing salts have been mined since the 19th century, resulting in numerous underground cavities, which have been filled with water since then (Schwandt und Seifert 1999), inducing a still ongoing leaching process.
The salt deposit has a layered structure (figure 1) where alternating more or less potassium bearing salt rock layers appear (Knak 1958). Since salt rocks of different composition shows different leaching characteristics, they necessarily have to be distinguished in a corresponding geometrical model.

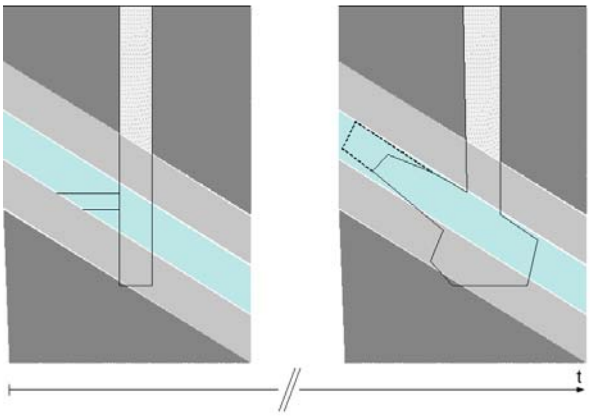

Figure 1 Salt leaching, Stassfurt, Germany
The left subfigure shows three different salt rock layer and the mining shaft, the right subfigure shows additionally the growing brine body.

Characteristic for potassium bearing salt is that not just salt is leached resulting in some kind of salty water (brine). In fact a circulation process occurs, while certain components become leached, others drop out (Sander 1988) and accumulate at a lower level, actually masking the leaching process in that area. The composition of the brine constantly changes over time while interactions constantly take place between salt rock and solution.
These dynamic interactions can be localized along the reaction surface between brine (fluid) and rock (solid), more basically between objects with different geochemical attributes. The direction and velocity of the solution process can be described by vectors, determined by an underlying process model, which integrates the relevant parameters of all involved objects (rock, fluid, reaction surface).

Thus, basic requirements for a geometrical model (the term model refers to the two- and three-dimensional geometrical models depicted in this paper) being

Proceedings 18th European Simulation Multiconference
Graham Horton (c) SCS Europe, 2004
ISBN 3-936150-35-4 (book) / ISBN 3-936150-36-2 (CD)

capable to represent the described features are as follows:

- many complex bodies
- dynamically altering objects
- differential geometry on the reaction surface
- interactions/interdependencies between objects
- topology preservation (no self penetration etc.)

The Geo Information System BAGIS and its successors (Kesper and Möller 1999), developed at the chair for Computer Engineering at the University of Hamburg, so far employed static parametric surfaces and solids (Körber et al. 2003), which are optimal for visualization and differential geometry. Since fundamentally altering parametric models is hard, as is preserving topology, the project group started to re-evaluate alternative dynamic solid models. This is recaptured on the following section. After that, the commonly used approach for dynamic geometric processes, physical based modelling, is considered with respect to the salt leaching process. This will be followed by a description of easily occurring, but hard to remedy topological entrapments. Finally an approach using a combination of voxel representation and parametric linear interpolation is suggested.

TYPES OF GEOMETRICAL MODELS

Geometrical Models can be partitioned into 5 broad classes (McDonnell 2000):

1. implicit Geometry
2. Constructive Solid Geometry (CSG)
3. parametric geometry
4. subdivision models
5. cell decomposition

Implicit Geometry defines geometry as the solution to an equation like $x^2 + y^2 - 1 = 0$ which defines the unit circle. This is, while mathematically exact, computational expensive and requires supportive algorithms such as the *marching cube* (Lorensen and Cline 1987) to be handled halfway efficiently. It is also very hard to find an implicit expression for complex geometry defined by samples. Finally, differential analysis is generally not possible on implicit functions unless converted to a parametric or an explicit form.

Constructive Solid Geometry (CSG) builds solids by applying set (Boolean) operations on primitive forms. This approach is strongly related to machines for metal processing for which it was invented. It is obviously unsuitable for geometry which is hard to combine from primitives and also inappropriate for direct differential analysis.

Parametric models are vector valued functions defining their shape by a regular grid of control points (Piegl and Tiller 1997). Initially developed for construction and especially for design (automotive engineering, aircraft construction and shipbuilding) Bézier-, B-Spline- and NURBS-models are meanwhile well elaborated tools. Beside shape modification, parametric models stand out by their differentiability due to the closed form basis functions (polynomials) of whom they consist. Nevertheless, we found it very hard to reconstruct real world geometry from samples, especially for solids, as known algorithms work only for surfaces since they require a projection onto a plane (Hormann and Greiner 2000) which is impossible for solids. Additionally, modifying the control point grid, e.g. to adapt it to increasing model complexity, is complicated and, especially in 3D, inefficient, since the regular grid structure must be maintained. Reconstruction is also hampered by the regular grid structure, which often does not fit well to asymmetric and complex geometry.

While the former three classes have a mathematical form, the remaining two are based on algorithms.

Subdivision models (Catmull and Clark 1978) are based on the successive refinement of an initial arbitrary grid. They converge, dependent on their subtype, to biquadratic resp. bicubic B-Spline representations. Because they lack basis functions, they don't share the mathematical abilities of parametric models. Thus, computing differential properties such as the derivatives, requires an indirection. Unlike parametric models, subdivision models don't depend on a regular control point structure, which makes them suitable for complexer and asymmetric geometry. Because of the structure of the algorithm they also have built-in level of detail (LOD) abilities (Hoppe 1998).

Cell subdivision finally completely discretizes the space into small regular units. Most common subtypes are voxel and octrees. Voxel decompose space into regular units, usually cubes. They obviously require n^3 space which is their greatest drawback, limiting model size and complexity more than any other model to the amount of available memory. Another suboptimal property is rendering which usually requires determining the boundary of the modelled object. Both can be overcome by employing another cell decomposition subtype, octrees, who decompose space hierarchically and uses fine resolution only for the objects boundary, allowing fast access to the boundary and decreasing memory consumption. Cell decomposition is commonly used in medical science, e.g. for magnetic resonance tomography (MRT) when the inside of an object is at least as interesting as its boundary. Both voxel and octrees suffer necessarily from their discrete structure, which can be very obvious, depending on the scale.

PHYSICAL BASED MODELING

The motivation of the attachment of a physical model to a geometrical model and thus deforming geometry dynamically, was derives from two reasons. One is to make models easier to handle, controlling them by forces the user experiences in everyday life. The other reason is to actually model a physical process on spatial objects.

The several degrees of freedom (control points, knot vectors, weights) parametric models provide, complicate controlled model modification, which is especially significant for NURBS. This resulted in the demand to apply real physical forces, e.g. pressure, on the model in order to achieve the desired deformation. (Terzopoulos 1987). Another demand came e.g. from medical sciences, asking for realistically behaving models which could be employed in real time for surgery training (Wu et al. 2001).

This behaviour can be achieved by employing Finite Element Methods (FEM) on parametric and subdivision models. This approach is called physical based modelling and is quite self-evident, since the geometric structure (triangular resp. quadrilateral grid) of the models corresponds well with a FEM polygon mesh generation.

However, the underlying physical laws are based on mechanics, i.e. the model is deformed based on e.g. pressure and tractive forces and using material properties like viscosity and elasticity.

Hence models can obviously be deformed by physical based modelling, but not altered in a more fundamental manner.

The salt leaching process is rather taking away something (the salt) from an object (salt rock) and adds it to another object (brine), than deforming objects. The salt leaching process could perhaps be mimicked by one object (the brine) applying pressure on another (salt), but this would require successively increasing forces in order to increasingly compress the salt body and feels generally inappropriate.

PRESERVING TOPOLGY

If the model of an object consists in fact of several smaller models, like the salt leaching area, topology problems my arise once that the carefully constructed model becomes subject of forces whose exact impact on the model is either unknown or not efficiently pre-computable. That means that preserving the topology of a robot arm, composed from several parts, is controllable, because all parameters are known and user or computer controlled, which makes it easy to detect e.g. self penetrations or even better, to prevent them in the first place. Figure 2 on the other hand illustrates exemplary cases which may arise if even simple 2D polygon models start to change their shape according to an underlying complex process. The figure shows schematically the reaction front between two salt layers and the brine body. One of the salt layers is hard and one is easy to leach which results in different leaching rates, which prevents to treat the layers as one. The two top subfigures show how the top layer penetrates the lower layer because the movement of the reaction front, induced by the leaching process, is modelled as the compression of a polygon model, see the preceding section. The lower subfigures show how insufficient area coverage may result from a geometry which becomes increasingly complex. The brine body in the lower right subfigure does not have enough vertices and edges to fill the space which results from the leaching.

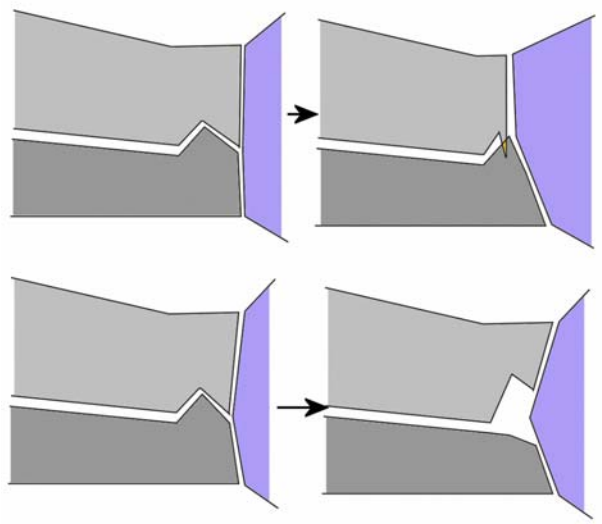

Figure 2 Illegal Topology due to Dynamic Geometry
top and bottom left: initial position, top right: self-penetration, bottom right: insufficient area coverage

Such issues could be partially overcome, though often for the price of inducing constraints and limitations. In the first example e.g. vertices could be moved along edges. This would require a regular grid to work, which is quite a constraint as described in the preceding sections.

CONCLUSION AND PERSPECTIVE

Non of the evaluated geometrical models optimally meet the requirements of the salt leeching process. While implicit geometry and CSG never really were candidates, also subdivision and parametric models come at a high price. It appears questionable whether the easily differentiable structure of parametric models or the arbitrary grid structure of subdivision models, justify the hassle expected from maintaining legal topology due to dynamic topology.

That brings cell decomposition into the focus, which was originally declined because this approach doesn't fit to BAGIS' Data Model (Kesper 2001), which would have to be extended.

Nevertheless, cell decomposition fits well to the hydro-geochemical process as one cell can simply switch attributes from salt to brine without bringing topology into any trouble. One issue which had to be dealt with is that the reaction surface moves very slow, perhaps 1cm per cycle of the underlying process model, which would then be the required resolution for e.g. voxel. We currently favor a model which combines cell decomposition and parametric properties by linking attributes not to voxel but to a regular grid of control points between whom we linearly interpolate. This allows a finer transition between control points / voxel without requiring more memory. Formally this is a linear solid B-Spline but since the control points lie on a regular grid, and the geometry thus implicit, the similarities to voxel are obvious. First test in 2D seem to confirm our expectations. Figure 3 shows a mimicking (no process model is used) of the salt leaching process, which does not show the hard edges which are typical for voxel.

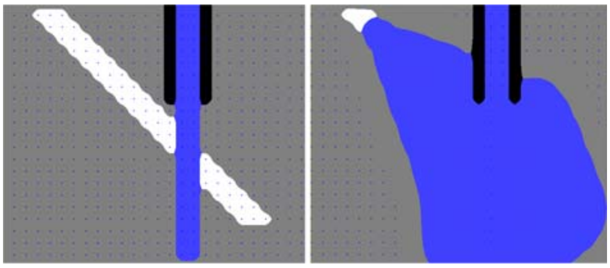

Figure 3 bilinear interpolating 2D cell decomposition of the investigation area

Some issues, like embedding several objects in one geometrical model, identifying the reaction surface and deriving its differential properties still need to be handled, but are considered easier to be handled than the mentioned topological and process related disadvantageous properties other models imply.

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