ANALYSIS OF DYNAMIC PROPERTIES OF AN INVENTORY SYSTEM WITH SERVICE-SENSITIVE DEMAND USING SIMULATION

Yuri Merkuryev and Julija Petuhova
Department of Modelling and Simulation
Riga Technical University
1 Kalku Street
LV-1658 Riga, Latvia
E-Mail: merkur@itl.rtu.lv, julija@itl.rtu.lv

Janis Grabis
Department of Operation Research
Riga Technical University
1 Kalku Street
LV-1658 Riga, Latvia
E-Mail: grabis@itl.rtu.lv

KEYWORDS
Inventory management, service-sensitive demand, hybrid modelling.

ABSTRACT

Complexity of many problems solvable by analytical methods quickly increases under additional assumption. Inventory management under the service-sensitive demand is one of such practical problems. This paper considers application of the hybrid simulation/analytical approach for dealing with this problem. The appropriate closed loop model, that incorporates both simulation and analytical models, has been developed. It has been applied to study behaviour of the inventory system under the service-sensitive demand. The regression analysis conducted indicates that the service-sensitive demand causes substantial deviations of the provided service level from the target service level. The target service level, the demand variability and the lead time are factors substantially influencing the difference between the target service level and the provided service level. The results obtained are to be used for design of a mechanism for adjusting the parameters of the inventory system in order to maintain the target service level.

INTRODUCTION

The comparative advantages and disadvantages of analytical versus simulation models are well known (for instance, see Nolan and Sovereign 1972). Hybrid simulation/analytical models are used to attain some of the advantages of both types of models, while avoiding the disadvantages. Shanthikumar and Sargent (1983) identify four classes of hybrid simulation/analytical models. Simulation and analytical models are independent parts of a model for the first class of hybrid models. These models are used sequentially. The second class includes hybrid models consisting of simulation and analytical models operating in parallel. The third class comprises hybrid models with dominant analytical models, which use subordinate simulation models for performing special tasks. Finally, in the fourth class a simulation model is a primary model of the system, and it uses inputs from one or more secondary analytical models during the modelling process. Such utilization of analytical models is frequently encountered in complex simulation models. For instance, analytical models are used to generate demand forecasts (Bhaskaran 1998) and for inventory management (Ganeshan et al. 2001). In order to simplify usage of analytical models in simulation, Baker (1997) develops a methodology for incorporating classic algorithms of operations research into simulation models.

This paper considers a special case of the fourth class hybrid simulation/analytical models. This case describes a situation, where a simulation model is built around an analytical model in order to extend functionality of the analytical model. This type of combination of analytical and simulation models is used to evaluate analytical models under realistic conditions and to consider model and environmental parameters not represented in the traditional formulation of analytical models. Cerda (1997) uses simulation to select the most appropriate ordering options for the complex multi-item re-order point inventory management policy. Clay & Grange (1997) simulate the supply chain of automotive service parts. The simulation model is used to evaluate the impact of different forecasting methods on the supply chain performance. Such analysis provides means for direct evaluation of forecasting methods by evaluating a resulted value of a specified goal function or variable (e.g., a service level) instead of evaluation of forecasting methods according to the forecasting accuracy criterion. Enns (2002) investigates the impact of forecasting accuracy on efficiency of materials requirements planning. In order to overcome limitations of previous research in this area, the author develops a shop floor simulation model for more realistic evaluation of elaborated production schedules. The application of the realistic production schedule evaluation procedure has enabled identification of complex interaction among properties of demand forecasts and characteristics of demand process. Takakuwa & Fujii (1999) develop a standardized simulation model for analysis of transhipment inventory systems. The simulation model is used to provide a more realistic representation of the transhipment problem comparative to the traditional mathematical programming representation. For model building purposes, the authors identify and standardize modules defining the transhipment problem and parameters of these modules.

Proceedings 15th European Simulation Symposium
Alexander Verbraeck, Vlatka Hlupic (Eds.)

ISBN 3-936150-28-1 (book) / 3-936150-29-X (CD)

A modelling problem considered in this paper is inventory management under service-sensitive demand. The service-sensitive demand implies that demand for future periods depends upon the service level observed at the current period. A traditional formulation of analytical inventory models does not consider such dependence.

The existing research on inventory management under service-sensitive demand has been restricted to either situations with deterministic demand or two-period problems (Baker and Urban 1988; Ernst and Powell 1995). These limitations can be explained by an explicitly dynamic character of the problem leading to a complicated analytical analysis. Simulation modelling allows analysing a multi-period problem under stochastic demand. The demand parameters change from one period to another in the case of the multi-period problem. Such behaviour can be observed in highly dynamic and competitive inventory systems. For instance, wholesalers of computer chips often are not able to meet demand due to insufficient supplies form upstream supply chain levels. In the case of the shortage, customers are likely to seek alternative vendors and may choose to place orders to a newly selected vendor for following periods as long as the service level is maintained. The customers may switch back to the initial vendor, if the newly selected vendor reduces its service level. Similar relationships between demand and performance of the inventory system are also observed in retail (Silver and Peterson 1985). The research on service-sensitive demand also relates to research on inventory level dependent demand (see Chung (2003) for a recent account).

Main objectives of this research are to expand modelling of inventory systems with service-sensitive demand by considering multi-period stochastic problems, identify properties of such systems and to test ability of traditional inventory models to meet service level requirements in the case of service-sensitive demand. The service level sensitive demand is modelled similarly to Ernst and Powell (1995). A simulation model is built around analytical models used for inventory management and updating demand parameters according to the observed short term service level. Experimental studies are conducted with the model, and the regression analysis is used to determine impact of service-sensitive demand on performance of the inventory system. The remaining part of the paper discusses issues in inventory management under service-sensitive demand, describes a simulation model for evaluation of such inventory systems and provides preliminary experimental results.

INVENTORY SYSTEM

A single-item, single-stage, multi-period inventory system is considered. The traditional re-order point policy is used for inventory management. The order size is fixed independently of the re-order point level.

The service level is measured using a proportion of demand satisfied directly from the inventory. Any unsatisfied demand is lost. External demand is normally distributed with mean $\bar{D}_t$ and the standard deviation σ_t, where $t = 1, ..., T, ...$. The parameters of the external demand change dynamically according to short-term fluctuations of the service level provided.

The inventory management objective is to preserve the service level at a fixed level. Alternatively, one could try to increase his or her market share. However, the market is assumed to be highly competitive, and other players are expected to take analogous actions preventing one player to achieve a permanent increase of the market share.

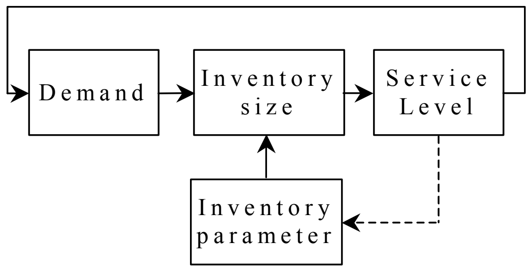

Figure 1: Interactions among Demand, Inventory Size and Service Level

Relationships between the demand, service level and inventory parameters are shown in Figure 1. The external demand causes depletion of the inventory level. The inventory is replenished according to the re-order point policy specified by a set of the inventory parameters (re-order point, order size, mean demand, demand standard deviation, lead time and target service level). The service level achieved during a relatively short time period is observed. This service level is most likely to differ from the target, required service level. In the case of the service-sensitive demand, this causes changes of the demand parameters. A higher than target service level causes increase of the mean demand and the standard deviation. A lower than target service level causes decrease of the mean demand and the standard deviation. The increase of the demand parameters may results in a declining service level in forthcoming periods unless the inventory parameters are properly adjusted. The decrease of the demand parameters may cause overstocking unless the inventory parameters are properly adjusted. Therefore, a link representing updating of the inventory parameters according to the observed short-term service level should be established.

SIMULATION MODEL

The inventory system described above has an explicitly dynamic character. Simulation is used to capture this behaviour of the system. A simulation model developed describes the inventory system and incorporates an analytical model for implementing a feedback between the simulated short-term fluctuations in the service level and the customer demand parameters. A structure of the considered hybrid simulation/analytical model is given in Figure 2.

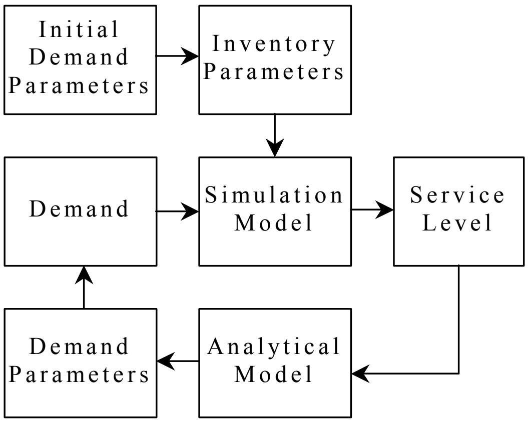

Figure 2: Structure of the Hybrid Simulation/Analytical Model

Simulation is used for analysis of properties of the system. Results of the analysis are expected to create a basis for developing a mechanism for updating of the inventory parameters in order to maintain the service at the target level.

Main steps of the simulation analysis being performed are as follows:

1. Initialise the input data module, including the initial (received by traditional forecasting techniques) forecast of the customer demand;
2. Perform simulation of inventory control processes;
3. Calculate the observed service level for a current time period;
4. Calculate the customer demand distribution parameters based on the observed service level;
5. Update the demand parameters in the simulation model;
6. Go to step 2 until simulation is completed.

Inventory management is based on the re-order point policy, where the order size Q is fixed independently of the re-order point level. The re-order point level is calculated using a formula:

$$ROP = LT * \overline{D}_0 + z * \sqrt{LT} * \sigma_0 , \qquad (1)$$

where LT is the lead time, $\overline{D}_0$ is an initial mean demand during one period, σ_0 is an initial standard deviation of the demand during one period, z is a safety factor that depends on a specified target service level.

The short term service level is calculated each period using a formula:

$$SL_t = 1 - \frac{SO_t}{D_t} , \qquad (2)$$

where t is a current time period, SO_t is an unsatisfied demand in period t, D_t is an observed actual demand in period t.

An impact of the customer service level on a future customer demand is quantified similar to Ernst and Powell (1995). In this approach, a linear relationship between the service level and the demand parameters is assumed. This dependence is evaluated based on parameters estimated by experts. This means that the mean demand increases/decreases by α points if the change in the service level doesn't exceed a certain threshold:

$$\overline{D}_t = (1 + \alpha * (SL_t - SL_{t-1})) * \overline{D}_{t-1} , \qquad (3)$$

where SL_{t-1} is the short term service level in the previous time period, $\overline{D}_{t-1}$ is the mean demand of the previous time period, α is a coefficient of the change in mean demand with increased/decreased service level.

The standard deviation of the demand for the new demand level is expressed as a function of the parameters α, β and the standard deviation σ_{t-1} from the previous time period:

$$\sigma_t = \left[1 + \beta^2 \alpha (SL_t - SL_{t-1})\right]^{\frac{1}{2}} \sigma_{t-1} , \qquad (4)$$

where β is a coefficient of the change in standard deviation of demand with changed service level.

In case if the increase/decrease in the service level exceed a restricted constant the mean demand is calculated by a formula:

$$\overline{D}_t = (1 + \alpha * (SL_t - SL_{t-1}) * MaxChange) * \overline{D}_{t-1} , \qquad (5)$$

where *MaxChange* is a constant of the maximal change in the service level.

The standard deviation of the demand in this case is found by a formula:

$$\sigma_t = \left[1 + \beta^2 \alpha (SL_t - SL_{t-1}) * MaxChange\right]^{\frac{1}{2}} \sigma_{t-1} \qquad (6)$$

If the short term service level is equal to one for two consecutive periods and the demand parameters do not exceed their initial values, the demand parameters are updated using the following expressions:

$$\overline{D}_t = (1 + \frac{\alpha}{10}) * \overline{D}_{t-1} , \qquad (7)$$

$$\sigma_t = \left[1 + \frac{\beta^2 * \alpha}{10}\right]^{\frac{1}{2}} \sigma_{t-1} . \qquad (8)$$

If this restriction is not imposed, the system may settle for providing a high service level on expense of carrying excessive inventory.

The simulation model is developed using the ARENA simulation modelling environment. Evaluation of the service level and updating of the demand parameters are implemented using Visual Basic.

EXPERIMENTAL EVALUATION

Experimental Design

Objective of experimental studies is to determine the short term customer service level, to identify parameters of the inventory system influencing disagreement between the target and observed service levels and to evaluate changes of the customer demand parameters. Therefore, a set of experiments with a feedback from the simulation model to the analytical model, when the demand parameters are updated taking into consideration the observed service level (service-sensitive demand), is performed. Performance of the inventory system is evaluated under various factors such as initial end customer mean demand, signal to noise ratio, target service level, lead time, and order size coefficient (Table 1).

Table 1: Experimental Design

Factors	$\overline{D}_0$	*Signal to Noise*	*Target Service Level*	*LT*	*Q*
Values					
Min	50	2	0.9	2	1x*LT*
Max	250	10	0.99	6	2x*LT*

The *Signal to Noise* factor describes variability of the demand process. A value of this factor normally should be in range between 1 and $\overline{D}_0$. Given the value of $\overline{D}_0$, the initial standard deviation (σ_0) is found by a formula:

$$\sigma_0 = \frac{\overline{D}_0}{Signal\ to\ Noise}, \qquad (9)$$

where $\overline{D}_0$ is the initial mean demand.

The *Target Service Level* factor describes the required service level of the inventory system considered. The *LT* factor value corresponds to time between the order placement time and the order arrival. It is measured in days. Two values of the fixed order size are considered. The minimum value is equal to the initial mean lead time demand, the maximum value is equal to the double initial mean lead time demand. The short term service level is observed every 5 days. The demand parameters are re-evaluated at the same time interval. The difference between the target (SL_{Target}) and the provided ($SL_{Provided}$) service level is the main performance measure. The provided service level represents the overall system service level and is calculated at the end of each run by a formula:

$$SL_{Provided} = 1 - \frac{\sum_{t=1}^{T} SO_t}{\sum_{t=1}^{T} D_t}, \qquad (10)$$

where T is a replication length.

Experiments are conducted according to a factorial experimental design with resolution IV. This design consists of 16 experimental cells. The model was run for 5 replications. Each replication length is defined as 250 weeks and a warm-up period is 20 weeks. Thus, simulation results are independent from the empty-and-idle initial state; there is no predetermined starting and finishing point for a simulation run.

Experimental Results

The simulation results are summarized using the regression analysis. We analyse how a difference between target and provided service levels is affected by values of the experimental factors considered. The dependent variable Y is defined as (SL_{Target} -$SL_{Provided}$). Estimated coefficients of the regression equation and associated p-values are reported in Table 2.

Table 2: Results of the Regression Analysis

Independent variables	Coefficients	p-value
Constant	-0.795	0.00
$\overline{D}_0$	0.00002	0.05
Signal to Noise	-0.004	0.00
LT	0.006	0.00
Order Size Coefficient	0.003	0.15
Target Service Level	0.848	0.00

Ideally, the difference between SL_{Target} and $SL_{Provided}$ should be equal to zero. However, the service-sensitive demand causes deviation of the provided service level from the target service level. Values larger than zero indicate that the required, target service level is not reached. Values smaller than zero mean that reached service level is higher than expected. The regression equation suggests that the *Signal to Noise* factor, *LT* and *Target Service Level* have the most significant impact on the difference between the target and provided service levels. The order size does not have the significant impact on this difference. The provided service level is likely to be smaller than the target service level, if the target service level is high, the lead time is long and the demand is highly variable. With a high degree of confidence (95%) the observed service level averaged over all experimental cells differs from the required service level by 5%, and the mean customer demand differs from the initial mean demand on average by 13%.

Dynamic behaviour of the mean demand and the observed short term service level during simulation is shown in Figures 3, 4, and 5. The results are obtained for different values of the target service level. Values of the initial mean demand, the signal to noise ratio, the lead time and the order size coefficient are fixed at 250, 2, 2, 1, respectively.

Figure 3: Mean Demand with Target Service Level 0.9

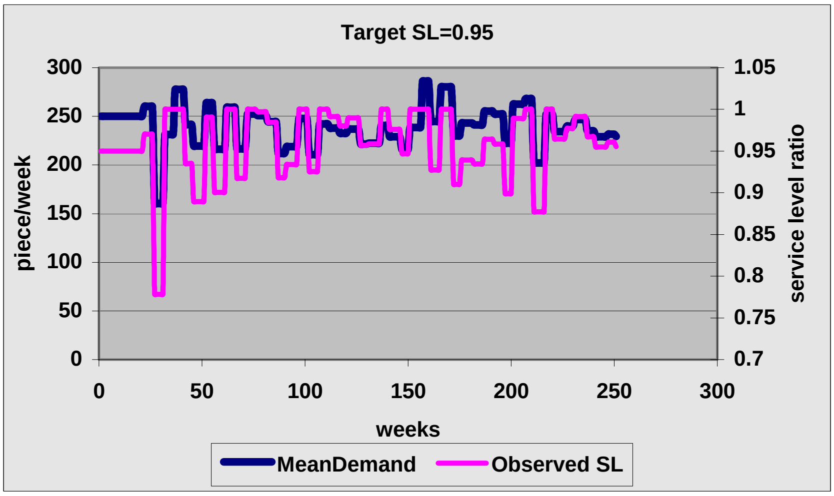

Figure 4: Mean Demand with Target Service Level 0.95

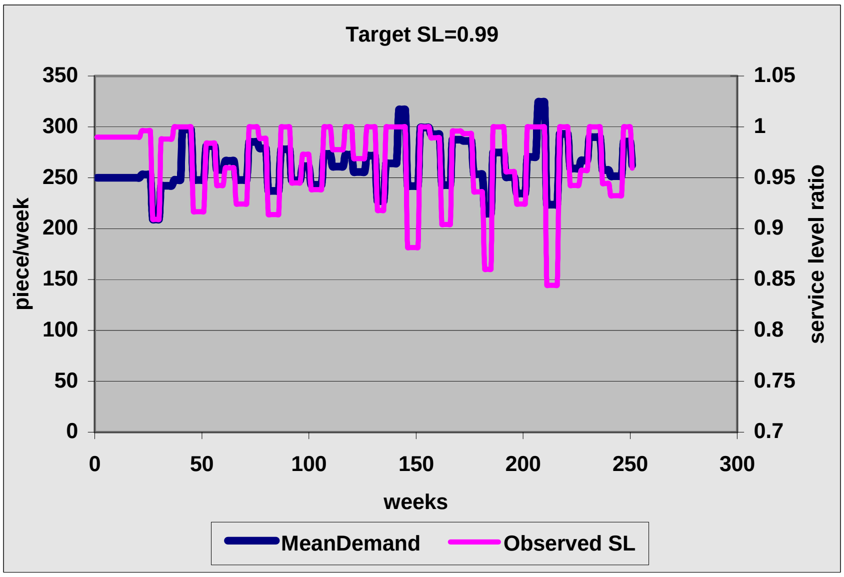

Figure 5: Mean Demand with Target Service Level 0.99

Experimental results show that the observed service level achieved each week often differs from the target service level. This leads to changes in the end customer demand. The demand volume changes are the main cause of the decreasing/increasing of the service level, because of reaching a high service level the demand volume become large and inventory control system is not able to adapt to a new environment during the short time. This leads to a lower service level in the next period and the demand volume becomes smaller. That increases the service level in the next period. This sequence is kept during a simulation run.

The target service level has an impact on the demand parameters. The demand parameters increase, if the target service level is high. The demand parameters decrease, if the target service is low. The increase of the demand parameters is observed because, in the case of the high target service level, the observed short term service level is often equal to one, shortages occur less frequently causing fewer possibilities for the demand parameters to decline.

CONCLUSION

The analysis of re-order point inventory systems under the service-sensitive demand has been extended to multi-period, stochastic demand situation. The hybrid simulation/analytical modelling approach has been advocated as an appropriate technique for conducting this analysis. The appropriate simulation model, which incorporates analytical models for inventory management and modelling of the service-sensitive demand, has been developed. It has been applied to study behaviour of the inventory system under the service-sensitive demand. Analytical models provide a well-defined mechanism for inventory management. However these models not give an impression of the system operation over the time. Therefore, a simulation technique is used to perform the analysis of the system dynamic behaviour.

The regression analysis conducted indicates that the service-sensitive demand causes substantial deviations of the observed service level from the required, target one. Additionally, the observed service level is lower than the target level, if latter is larger, while the observed service level is higher than the target one, if latter is smaller. The demand variability and the lead time are other factors substantially influencing the difference between the target and the provided service levels. The results obtained are to be used for design of a mechanism for adjusting the parameters of the inventory system in order to maintain the target service level. The simplest mechanism for achieving this objective is recalculating of the inventory parameters according to new values of the demand parameters. However, preliminary studies of this mechanism suggest that a more complex preemptive approach is needed.

REFERENCES

Baker, G.S. 1997. "Taking The Work Out Of Simulation Modeling: An Application Of Technology Integration". *In Proceedings of the 1997 Winter Simulation Conference*, 1345-1351.

Baker, R.C.; and Urban, T.L. 1988. "A deterministic inventory system with an inventory-level-dependent demand rate". *Journal of the Operational Research Society* 39, 9, 823-831.

Bhaskaran, S. 1998. "Simulation analysis of a manufacturing supply chain". *Decision Sciences* 29, 3, 633-657.

Cerda C.B.R.; and de los Monteros, A.J.E. 1997. "Evaluation of a (R,S,Q,C) Multi-Item Inventory Replenishment Policy Through Simulation". *In Proceedings of the 1999 Winter Simulation Conference,* 825-831.

Chung, K.-J. (2003), An algorithm for an inventory model with inventory-level-dependent demand rate, *Computers & Operations Research* 30, 9, 1311-1317.

Clay, G.R.; and Grange, F. 1997. "Evaluating Forecasting Algorithms And Stocking Level Strategies Using Discrete-Event Simulation", *In Proceedings of the 1997 Winter Simulation Conference,* 817-824.

Enns, S. T. 2002. "MRP performance effects due to forecasting bias and demand uncertainty". *European Journal of Operational Research* 138, 87-102.

Ernst, R. and S. G. Powell (1995), Optimal inventory policies under service-sensitive demand, *European Journal of Operational Research* 87, 316-327.

Ernst, R. and S. G. Powell (1998), Manufacturer incentives to improve retail service level, *European Journal of Operational Research* 104, 437-450.

Ganeshan, R., T. Boone and A. J. Stenger (2001), The impact of inventory and flow planning parameters on supply chain performance: An exploratory study, *International Journal of Production Economics* 71, 1-3, 111-118.

Ho, P.-K. and J. Perl (1995), Warehouse location under service-sensitive demand, *Journal of Business Logistics* 16, 1, 133-162.

Nolan, R. L. and M. G. Sovereign (1972), A recursive optimization and simulation approach to analysis with an application to transportation systems, *Management Science* 18, 12, 676-690.

Shanthikumar, J. G. and R. G. Sargent (1983), A Unifying View of Hybrid Simulation/Analytic Models and Modeling, *Operations Research* 31, 6, 1030-1052.

Silver, E.A.; and Peterson R. 1985. Decision systems for inventory management and production planning, 2nd ed. New York: Wiley.

Takakuwa, S.; and Fujii, T. 1999. "A Practical Module-Based Simulation Model for Transshipment-Inventory Systems". *In Proceedings of the 1999 Winter Simulation Conference,* 1324-1332.