MATHEMATICAL MODELLING AND IDENTIFICATION OF THE FLOW DYNAMICS IN MOLTEN GLASS FURNACES

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ABSTRACT

In the paper a new method for computer aided modelling and identification of flow dynamics in molten glass is presented and numerically analysed. The process of the construction of the model occurs in several steps on which the sub-models with differentiated mathematical descriptions (with distributed or lumped differential equations) and dynamical features (with inertial and oscillatory characteristics and with slow and fast changeable dynamics) are setting up. This method makes possible to prepare the models of glass tank furnaces of high degree of accuracy, described with the equations of high orders. The models are suited well to estimate technological parameters of glass tank furnaces and to control the glass melting process.

INTRODUCTION

The glass production is a very complex technological process. For this reason there is very difficult to set up its mathematical models that could be useful for practical applications, such as computer simulation, control or estimation of technological parameters. The modelling of glass tank furnaces occurs usually under separated application of Distributed or Lumped Parameter Equations (DPE or LPE models) and the result is that they are very complicated (DPE models) or very simplified (LPE models). That is why their practical usefulness is very limited. This situation is caused by lacking of adequate identification methods. Thus in the following a numerical algorithm is presented for setting up molten glass models under consideration of both arts of mathematical description. This way their drawbacks could be eliminated and their advantages retained. The algorithm presented consists of two general stages. On the first stage a DPE model is formulated with the quasi-linear Navier-Stokes and energy equations and with an equation added that describes the glass mass composition change in the molten glass. On the second stage a complex LPE model is prepared using the DPE model previously identified. All computations are done using real data from a industrial glass tank furnace. The fitting of the DPE model to the data occurs by using static optimisation methods. To estimate the structure and the parameters of the LPE model an indirect identification method is used, developed especially for setting up continuous dynamic models of higher orders.

DPE MODEL FORMULATION

To model the glass mass flow in a tank furnace by the partial differential equations the following description is used (Studzinski 2002a):

$$\begin{cases} \mu(T)(D_1^2 v_1 + D_2^2 v_1) = D_1 p \\ \mu(T)(D_1^2 v_2 + D_2^2 v_2) = D_2 p - \rho g \beta (T - T_o) \end{cases} \tag{1}$$

$$\lambda(T)(D_1^2 T) + \lambda(T)(D_2^2 T) = \rho c_v (v_1 D_1 T + v_2 D_2 T) \tag{2}$$

$$D_1 v_1 + D_2 v_2 = 0 \tag{3}$$

$$\frac{\partial z}{dt} + e_1 v_1 \frac{\partial z}{\partial x_1} + e_2 v_2 \frac{\partial z}{\partial x_2} = D(T)(e_3 \frac{\partial^2 z}{\partial x_1^2} + e_4 \frac{\partial^2 z}{\partial x_2^2}) \tag{4}$$

where the parameters mean: v_1, v_2 – longitudinal and vertical glass mass velocities in x_1 and x_2 directions, p – pressure, T – temperature and T_o – reference temperature, z – chemical composition of the melt, t – time, μ – dynamic viscosity, ρ – density, g – gravitational acceleration, β – thermal expansion, λ – thermal conductivity, c_v – specific capacity, D – diffusion coefficient, e_1 to e_4 – some fitting coefficients (to fit the model to an object).

Equations (1), (2), (2) are known in the classical fluid mechanics as the Navier-Stokes (or motion), energy and continuity equations, respectively, and they are formulated on the base of the momentum, energy and mass conservation laws. These equations describe the distributions of the temperature and the glass melt velocities in a tank furnace induced by the free and forced convections currents in the molten glass. Equation (4) describes the glass mass composition changes induced by the convection currents and the diffusion. While setting up the equations several simplified assumptions were made that took into consideration the specific properties of the glass mass flow and also the hypothesis that the glass melt is an incompressible and Newtonian liquid (Studzinski

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2002a). The scheme of a glass tank furnace modelled and the main convection currents occurring in the molten glass are shown in Fig. 1.

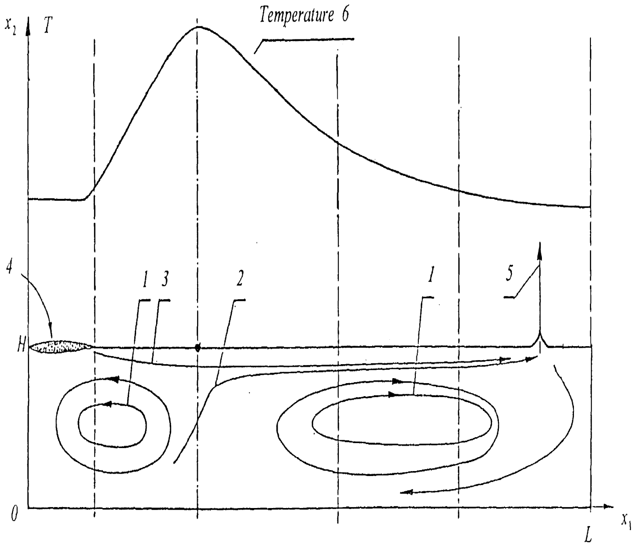

Figure 1: Longitudinal section of the glass tank furnace and the main currents occurring in the melt; 1,2,3 - rotating, withdrawal and surface current, respectively, 4 - raw materials input, 5 - glass take-out, 6 - temperature distribution on the free surface of the glass melt.

DPE MODEL IDENTIFICATION

Equations (1-3) make together a two-dimensional DPE model of the glass mass flow in a tank furnace. After some boundary conditions are given and the temperature and velocities values are calculated from equations (1-3), one can calculate subsequently the glass melt composition at each point of the tank by solving equation (4). To get the numerical solution of the model equations the finite difference method is used and a theoretical analysis of the numerical solvability of the model is made (Studzinski 2002a). On the first step of the model computing equations (1-3) are solved. The boundary conditions for the function *p* are unknown and this makes necessary to transform the equations. It is done by replacing the velocities v_1, v_2 by the current function ψ what results in a new model form consisting of only two equations contrary to the four ones in (1-3). The reduction of the number of equations causes in general a better convergence when solving the model numerically. A discrete approximation of the model equations occurs by the help of difference quotients. The use of standard difference quotients leads, however, in the case of high order derivations of equations to a bad stability of the resulted difference schemes at the edges of the knotted grid. To improve the approximation some new central difference quotients have been developed for the high order derivations of ψ. The difference schemes resulted from equations (1-3) are solved by means of the relaxation method using an iterative algorithm. For the numerical calculation the values of the physical coefficients and of the space dimensions of the model were chosen according to those ones of a real tank furnace. The convergence of the iterative algorithm was relatively fast with highly satisfactory accuracy of the calculation. Some results of the temperature and current fields computed are shown in Fig. 2. One can see in Fig. 2 that only the rotating and withdrawal currents but not the surface current (as it is shown in Fig. 1) are determined after the simulation of model (1-3) was made. This current could not be obtained with a two-dimensional DPE-model.

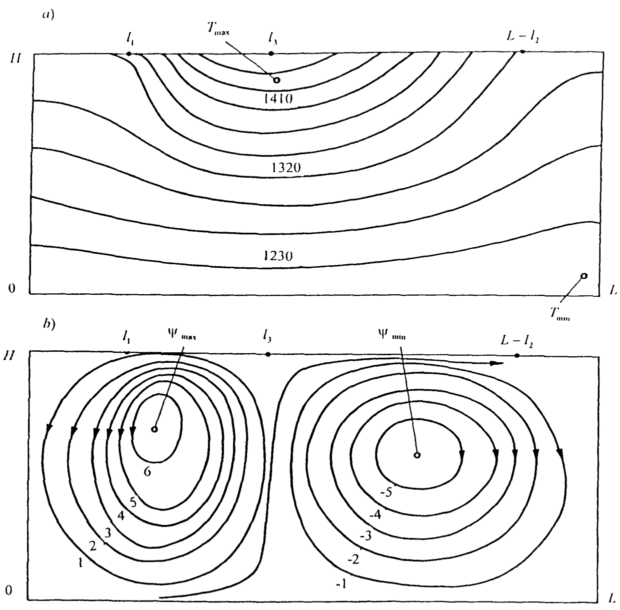

Figure 2: Computed temperature (figure a) and current distribution (figure b) in the glass melt for the longitudinal section of the glass tank furnace.

The numerical solution of equation (4) occurs on the second step of the modelling. To approximate (4) some central difference quotients of the finite difference method are used and as a result a new difference scheme with some fitting coefficients is obtained. The glass tank model described by equations (1-4) constitutes an approximation of a real object. Such an approximation is usually not exact although the parameters and dimensions of the model correspond to those ones of the tank. The possible inaccuracies occur while the model equations and boundary conditions are formulated and the parameter values are determined. Also the numerous simplifications made during the setting up the model are responsible for many inaccuracies and this is practically unavoidable. Then the fitting of the model to the object can be realised by the help of equation (4) and some measurements data obtained from the tank furnace under investigation (see Fig. 3). To do it the following identification problem is formulated:

$$\min_{e_1} Q(e_i) = \min_{e_i} \sum_{k=0}^{K} (\hat{z}^k - z^k)^2 \qquad (5)$$

where $\hat{z}^k$ and z^k mean the measured data and the discrete values of the model output that is calculated by solving equation (4) (the glass composition z is here considered as the radioactivity of the glass melt that has been measured while realising an isotope experiment on the tank furnace). To solve problem (5) a static non-gradient optimisation method is used (Studzinski 2002b). The criterion function $Q(e_i)$ is strong non-linear relating to e_i. Because of that the start points for the optimisation runs had to be chosen very carefully and close enough to the optimum. The model output obtained from the calculation is shown in Fig. 3. One can see that the output fits well to the data in the farther section of the curve where the influence of the rotating and withdrawal currents on the glass mass flow is the strongest. The approximation of the data with the model output in the initial section of the curve is much worse but there the surface current determines the data which is noticeable through the high oscillations of the curve. This situation can be explained through the omission of the surface current in the DPE model. This current could be considered in a three-dimensional DPE model but unfortunately such a model would be hardly possible to identify because of its great complexity.

LPE MODEL AND ITS IDENTIFICATION

The glass mass flow in a tank furnace can be described using also LPE models. Their parameters have no physical meaning and this gives interpretation troubles when comparing the models and objects. On the other side the setting up of such the models is easier than PDE models regarding the work complexity and the computing time needed for simulation and identification. Usually the non-linear regression methods are used for developing the lumped parameter models. These methods are generally successful if models of lower orders have to be set up but they are not effective in more complicated cases. The main problems then are connected with the choice of an adequate model structure and with the fixing a start point possibly closely to the optimum while making the identification. The methods of non-linear regression converge usually to the local optimal points if the start points are not right.

To overcome these problems an indirect identification method was developed to model linear dynamic objects of higher orders from their sampled impulse responses (Nahorski et al. 1985). This method has been adopted for setting up the LPE models of glass tank furnaces by using a multistage modelling approach (Studzinski 2002b). The mathematical description of an object modelled is now in the form of the homogeneous ordinary differential equation:

$$\frac{d^R z}{dt^R} + a_{R-1} \frac{d^{R-1} z}{dt^{R-1}} + \ldots + a_o z = 0 \qquad (6)$$

with the non-zero initial conditions added:

$$\begin{cases} z(0) = b_{R-1} \\ z^{(1)}(0) = b_{R-2} - a_{R-1} z(0) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ z^{(R-1)}(0) = b_o - a_1 z(0) - \ldots - a_{R-1} z^{(R-2)} \end{cases} \qquad (7)$$

and with the following analytical solution function:

$$z(t) = \sum_{j=1}^{J} \sum_{l=0}^{m_j - 1} t^l exp(\alpha_j t)\left(c_{jl} cos(\varphi_j t) + d_{jl} sin(\varphi_j t)\right)$$

$$\sum_{j=1}^{J} m_j = R \text{ and } m_j > 0 \qquad (8)$$

The continuous equation (6) can be approximated by the following discrete equation:

$$z_k + s_{R-1} z_{k-1} + \ldots + s_o z_{k-R} = 0 \qquad (9)$$

with: $z_k = z(k\Delta(k$, k=1,2,...,K, Δt – sampling step, and with the following analytical solution function:

$$z_k = \sum_{j=1}^{J} \sum_{l=0}^{m_j - 1} k^l \sigma_j^k \left(f_{jl} cos(\psi_j k) + g_{jl} sin(\psi_j k)\right) \qquad (10)$$

By comparison (10) and (8) one can convert very easy the coefficients of function (10) into the coefficients of (8).

The numerical algorithm realising the indirect identification method is as follows:

1. Fitting the difference equation (9) to the impulse response obtained from the object, using a standard time series identification method.
2. Estimation of the coefficients in the time discrete function (10) using a standard optimisation method (e.g. the linear regression) and the parameters identified in (9).
3. Calculation of the coefficients in the time continuous function (8) converting the coefficients of function (10) with the help of some simple algebraic formulas.
4. Calculation of the parameters of equations (6) with the help of the parameters of (8).

The main idea of the indirect identification method is that at first a discrete model is found and afterwards it is converted into the time continuous one. In this way the search for a continuous model is realised „indirectly", i.e. using a discrete model that is much easier to develop from the numerical point of view. In the case of complex objects it is well-advised to divide the modelling process into several stages at which sub-models with different dynamics features are constructed and afterwards put together to one overall model. On each stage of modelling different data sequences must be used for identification and they are to be isolated from the original measurements. The currents distribution occurring in the glass melt (see Fig. 1) suggests that the features of the melt mixing dynamics in a tank furnace depend in a different way on the character and velocities of the currents. The slow-running withdrawal current decides on the dynamics of the slow-varying inertial character and the fast-running surface current, as well as the rotating currents decide on the dynamics of the different-varying oscillatory characters. Also the isotope data for identification display both the inertial and oscillatory characters (see Fig. 3). The above remarks justify the application of the multistage approach for modelling glass tank furnaces. The choice of the best („optimal") sub-models as well as of the best overall model occurs by means of the residual sums.

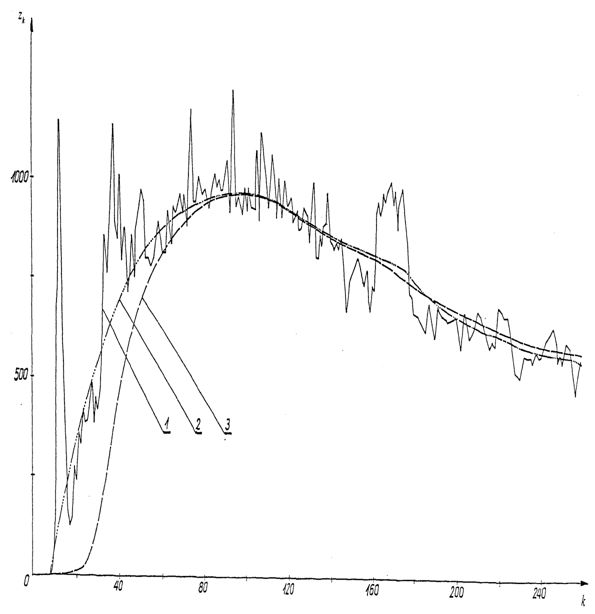

Figure 3: Isotope data for modelling the glass mass flow dynamics; 1,2- noisy and smoothed data, respectively, 3 - output of the DPE model.

Some models have been developed for the glass tank furnace under consideration using this multistage approach They fit well to the farther part of the data curve (where the „slow" dynamics of the object dominate) but their adaptation to the initial phase of the curve (where the oscillatory components dominate) is much worse. The modelling of this initial data section depends considerably on the division of the whole data sequence into the components which are used for setting up the sub-models. This makes the main trouble when using the multistage modelling approach with the LPE description of the models. Since the runs of the data components are not known from the beginning, they can be guessed only in general and the appropriate data curves are obtained using various smoothing algorithms. This leads, however, to great inaccuracies of the proceeding.

COMBINED ALGORITHM OF MOLTEN GLASS MODELING

To avoid the disadvantages of the above modelling approaches a combined algorithm for modelling glass tank furnaces has been developed. The final models obtained by means of this approach are described by ordinary differential equations but a DPE model is used at the first level of the modelling. The conception of this combining modelling resulted from the experience which was gathered after the models with distributed and lumped parameters were developed separately. In the latter case the modelling of the oscillations appearing in the isotope data (and caused by the surface current) is not exact. The only use of smoothing algorithms does not allow to determine exactly the initial run of the data curve which is used later to set up the „slow" inertial sub-model and because of that there is not possible to get the right data for farther stages of the modelling. But these difficulties can be surmounted by help of the DPE model. It allows to isolate correctly the surface current component of the data from the component which is responsible in the main for the glass mass transport in a tank furnace. This component is caused by the withdrawal and rotating currents and it is approximated correctly by the DPE model.

The two-level modelling approach is as follows:

1. Formulation of the partial differential equations describing the DPE model and its computer simulation.
2. Identification of the DPE model with the help of the measurements data and by means of an optimisation method.
3. Developing of the slow-varying LPE sub-model using the output of the DPE model as the data for the indirect identification method.
4. Preparation of the data for setting up the fast-varying LPE sub-model by subtracting the output of the DPE model from the original measurements and by smoothing the results (this sub-model will describe the contribution of the surface current in the measurement data).
5. Developing of the fast-varying LPE sub-model using the indirect identification method.
6. Putting together the sub-models into one overall model and the subsequent estimation of its parameters by means of the non-linear regression methods.

After using the combined modelling algorithm a complex LPE model of the glass mass flow dynamics was finally set up. The model has the eleventh order and it consists of two sub-models of sixth and fifth orders, respectively. The 6th order sub-model has the inertial-oscillatory character and owns two real and four complex roots in its transfer function. It fits very well to the output of the PDE model. The 5th order sub-model has either the inertial-oscillatory character and it has one real and four complex roots in its transfer function. It fits very well to the oscillations caused by the surface current. The overall LPE model fits well to the original measurements and it approximates exactly the oscillations occurring in the initial section of the data (see Fig. 4).

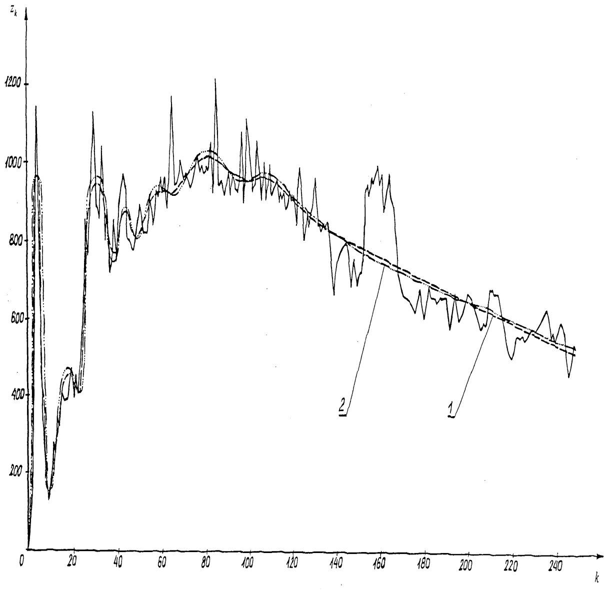

Figure 4: Overall LPE models obtained by means of the combined modelling approach without and after using the non-linear regression method (curve 1 and 2 respectively).

CONCLUSIONS

The problem of mathematical modelling of the glass mass flow dynamics in a glass tank furnace is solved and three numerical approaches of modelling are presented, tested and discussed. The first approach develops two-dimensional DPE models that describe the slow-varying dynamics of the glass tanks in which the withdrawal and rotating currents occur and no surface current appears. The second approach allows the development of LPE models of relatively small order that do not describe exactly the complex dynamics of the objects in which all kinds of the currents occur. The troubles arise while modelling by means of this approach the initial section of the isotope data where the simultaneous effects of the slow and fast varying currents are particularly strong. There is no effective algorithm to divide the data curve into the components for there is not known *a priori* in which way the individual currents influence the measurements. The third approach is a combination of the two and it makes possible to develop complex LPE models of the high order that have got the inertial-oscillator features and very differentiated parameter values.

The models computed by help of this approach describe very well the dynamics of the glass mass flow and all the same they are simple and convenient enough for numerical treatment. They can be used for the development of control or stabilisation algorithms with reference to the chemical composition of the glass as well as for the calculation of the technological parameters of glass tank furnaces.

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