# The Implementation of CGF-Oriented Helicopter Dynamic Model 

Jianxiang Liu<br>Haozhi Li<br>Beijing Institute of System Engineering<br>P. O. Box 9702-19\#<br>Beijing, 100101 China

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#### Abstract

CGF(Computer Generated Forces) means a set of virtual entities which can be run autonomously in the distributed virtual battle field. The physical behaviors incarnating their motion are the basis of other higher intelligent behaviors, so the entities' dynamic model should be built. In this paper a generally simplified helicopter dynamic model and control system model is implemented, it has been applied to the simulation of CGF to change the entities' motion status, which can reduce the calculation complexity when the precision is not affected and prevent singularity from occurring by quaternion.


## 1 INTRODUCTION

Distributed Virtual Battle Field Environment (DVBFE) is becoming the next generation infrastructure of military training. In DVBFE, the operators of all kinds of weapon system simulators can collaborate and perform tactics rivalry through the network. In order to enrich the battle field environment and improve its verisimilitude, a lot of computer-generated virtual weapon entities called CGF run autonomously in distributed virtual battle field to provide rivals for the people involved in the rivalry, CGF entities' physical behaviors that incarnate their motion are the basis of other higher intelligent behaviors. So it is needed to build a model for their physical behaviors. Modeling for the virtual weapon system entities is mainly related to their dynamics and their control systems.
According to the characteristic of CGF, a generally simplified helicopter dynamic model is implemented in this paper, and an integral method is explored for the model when it is discretized. In addition, during the calculation, the occurrence of singularity is prevented by quaternion.

## 2 MODELING OF HELICOPTER DYNAMICS

To simplify the model of the helicopter dynamics, this paper only reckons in the air dynamic power and moment affecting the rotors, airframe and empennage of helicopter, and ignores the dynamic power affecting the other vanes
and the ground effect when the helicopter flights in low altitude. As shown in figure 1, Earth Axes OdXdYdZd and Aircraft Axes ObXbYbZb are defined, where Od is a random point on the ground, Xd points to the east, Yd points to the north, according to the right-hand rule, Zd points to the vertical down; Ob is barycenter of the airframe, Xb points to the nose of the helicopter, Yb points to the right of the airframe, Zb points down.


Figure 1 The Definition of Earth Axes and Aircraft Axes
In Aircraft Axes, the effects of the air dynamic power and the moment on the helicopter are:

$$
\begin{equation*}
\vec{F}_{b}=\stackrel{\rightharpoonup}{F}_{b}^{R}+\stackrel{\rightharpoonup}{F}_{b}^{f}+\stackrel{\rightharpoonup}{F}_{b}^{T R} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\vec{M}_{b}=\vec{M}_{b}^{R}+\vec{M}_{b}^{f}+\vec{M}_{b}^{T R} \tag{2}
\end{equation*}
$$

where, $R, f$ and $T R$ represent the rotors, airframe and empennage respectively. Thus the power and the moment of the rotors and empennage can be calculated by Blade Element Approach. In addition, the power affecting the airframe $\vec{F}_{b}$ is simplified to a function of the velocity $\vec{V}$, i.e. $\vec{F}_{b}=G(\vec{V})$. The inputs of dynamics system are the collective pitch, the rotor longitudinal feathering, the rotor lateral feathering and the tail rotor pitch. (Please refer to [2] for the concrete calculation).
Assuming that $\mathrm{u}, \mathrm{v}$ and w are the speed values of the helicopter velocity $\vec{V}$ in three axes respectively, we can get:

$$
\left\{\begin{array}{l}
\dot{u}=r v-q w-g \sin \theta+\frac{F_{x}}{m}  \tag{3}\\
\dot{v}=p w-r u+g \sin \phi \cos \theta+\frac{F_{y}}{m} \\
\dot{w}=q u-p v+g \cos \phi \cos \theta+\frac{F_{z}}{m}
\end{array}\right.
$$

where, $F_{x}, F_{y}$ and $F_{z}$ are the power value of the three axes respectively, $\theta, \psi$ and $\varphi$ are pitch, yaw and bank respectively.
When calculating the attitude of the helicopter, we only reckon in the moment value of inertia in the three axes. Assuming that $\mathrm{p}, \mathrm{q}$ and r are the angular rates of $\mathrm{X}_{\mathrm{b}}$, $Y_{b}$, and $Z_{b}$ respectively, then it can be obtained:

$$
\left\{\begin{array}{l}
\dot{p}=\frac{M_{x}}{I_{x}}  \tag{4}\\
\dot{q}=\frac{M_{y}}{I_{y}} \\
\dot{r}=\frac{M_{z}}{I_{z}}
\end{array}\right.
$$

where, $I_{x}, I_{y}$ and $I_{z}$ are the moment values of inertia in the three airframe axes respectively, and $M_{x}, M_{y}$ and $M_{z}$ are the moment values of the three axes respectively.
According to the Euler equation, the three attitude angles $\theta, \psi$ and $\varphi$ can be calculated by

$$
\left[\begin{array}{c}
\dot{\phi}  \tag{5}\\
\dot{\psi} \\
\dot{\theta}
\end{array}\right]=\left[\begin{array}{ccc}
1 & -\cos \phi \sin \theta / \cos \theta & \sin \phi \sin \theta / \cos \theta \\
0 & \cos \phi / \cos \theta & -\sin \phi / \cos \theta \\
0 & \sin \phi & \cos \phi
\end{array}\right]\left[\begin{array}{l}
p \\
q \\
r
\end{array}\right]
$$

As shown in the equation (5), when the pitch value $\theta$ is $90^{\circ}$, i.e. $\cos \theta=0$, the angles cannot be calculated due to the occurrence of singularity. To avoid this case, a coordinate transition through quaternion is made as follow.
Assume $\mathrm{B}_{\mathrm{db}}$ is the transition matrix from Aircraft Axes to Earth Axes, that is:

$$
B_{d b}=\left[\begin{array}{lll}
l_{11} & l_{12} & l_{13}  \tag{6}\\
l_{21} & l_{22} & l_{23} \\
l_{31} & l_{32} & l_{33}
\end{array}\right]
$$

where

$$
\begin{aligned}
& l_{11}=q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} \\
& l_{12}=2\left(q_{1} q_{2}-q_{0} q_{3}\right) \\
& l_{13}=2\left(q_{1} q_{3}+q_{0} q_{2}\right) \\
& l_{21}=2\left(q_{1} q_{2}+q_{0} q_{3}\right) \\
& l_{22}=q_{0}^{2}-q_{1}^{2}+q_{2}^{2}-q_{3}^{2} \\
& l_{23}=2\left(q_{2} q_{3}-q_{0} q_{1}\right) \\
& l_{31}=2\left(q_{1} q_{3}-q_{0} q_{2}\right) \\
& l_{32}=2\left(q_{2} q_{3}+q_{0} q_{1}\right) \\
& l_{33}=q_{0}^{2}-q_{1}^{2}-q_{2}^{2}+q_{3}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& q_{0}=\cos \frac{1}{2} \psi \cos \frac{1}{2} \theta \cos \frac{1}{2} \varphi+\sin \frac{1}{2} \psi \sin \frac{1}{2} \theta \sin \frac{1}{2} \varphi \\
& q_{1}=\cos \frac{1}{2} \psi \cos \frac{1}{2} \theta \sin \frac{1}{2} \varphi-\sin \frac{1}{2} \psi \sin \frac{1}{2} \theta \cos \frac{1}{2} \varphi \\
& q_{2}=\cos \frac{1}{2} \psi \sin \frac{1}{2} \theta \cos \frac{1}{2} \varphi+\sin \frac{1}{2} \psi \cos \frac{1}{2} \theta \sin \frac{1}{2} \varphi \\
& q_{3}=-\cos \frac{1}{2} \psi \sin \frac{1}{2} \theta \sin \frac{1}{2} \varphi+\sin \frac{1}{2} \psi \cos \frac{1}{2} \theta \cos \frac{1}{2} \varphi
\end{aligned}
$$

The speed values in the three axes of the helicopter can be calculated by

$$
\left[\begin{array}{c}
\dot{y}_{d}  \tag{7}\\
\dot{x}_{d} \\
\dot{z}_{d}
\end{array}\right]=B_{d b}\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]
$$

where $x_{d}, y_{d}$ and $z_{d}$ are the displacements of helicopter in $X_{d}, Y_{d}$ and $Z_{d}$ respectively.
The three attitudes are given by:

$$
\left\{\begin{array}{l}
\theta=\arcsin \left(2\left(q_{1} q_{2}-q_{0} q_{3}\right)\right)  \tag{8}\\
\phi=\arccos \left(\frac{1-2\left(q_{1}^{2}+q_{3}^{2}\right)}{\cos \theta}\right) \\
\psi=\arccos \left(\frac{1-2\left(q_{2}^{2}+q_{3}^{2}\right)}{\cos \theta}\right)
\end{array}\right.
$$

In our implemented system, the ranges of the attitudes are limited as:

$$
\begin{equation*}
-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad-\pi \leq \phi \leq \pi, \quad-\pi \leq \psi \leq \pi \tag{9}
\end{equation*}
$$

In addition, in order to calculate the quaternion, we define a middle matrix $\Omega$ as:

$$
\Omega=\left[\begin{array}{cccc}
0 & p & q & r  \tag{10}\\
-p & 0 & -r & q \\
-q & r & 0 & -p \\
-r & -q & p & 0
\end{array}\right]
$$

Thus the changing rate of the quaternion can be calculated by:

$$
\left[\begin{array}{c}
\dot{q}_{0}  \tag{11}\\
\dot{q}_{1} \\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]=-\frac{1}{2} \Omega\left[\begin{array}{l}
q_{0} \\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]
$$

and the quaternion can be calculated through integral when the equation is discretized.

## 3 CONTROL SYSTEM MODEL

Based on section 2 of this paper, the inputs of the dynamic model are the collective pitch, the rotor longitudinal feathering, the rotor lateral feathering and the tail rotor pitch, the power and the moment can be changed by these inputs. However, in the real operation, we cannot always get the
expected values immediately. These values must experience a transient process. In our system, we simulated the process through 1-order system, assume that the expected input increment is $\Delta \mathrm{Ue}$, and the factual input increment is $\Delta \mathrm{U}$, their relationship is:

$$
\begin{equation*}
\frac{\Delta U}{\Delta U_{e}}=\frac{1}{\tau s+1} \tag{12}
\end{equation*}
$$

Then $\Delta \mathrm{U}$ can be calculated by:

$$
\begin{equation*}
\Delta U=\Delta U_{e} \bullet\left(1-e^{-\frac{\Delta t}{\tau}}\right) \tag{13}
\end{equation*}
$$

## 4 NUMERICAL ALGORITHM OF THE MODEL

Based on the previous analysis, the output of the dynamics system can be expressed as six 1-order differential equations as follows:

$$
\begin{align*}
\dot{u} & =f_{1}(v, w, q, r) \\
\dot{v} & =f_{2}(u, w, q, r) \\
\dot{w} & =f_{3}(u, v, q, r) \\
\dot{p} & =f_{4}(q, r)  \tag{14}\\
\dot{q} & =f_{5}(p, r) \\
\dot{r} & =f_{6}(p, q)
\end{align*}
$$

In order to achieve high precision, the equation (14) are usually solved by the Runge-Kutta method, but it need much resources to calculate. In this paper, the dynamic equation of helicopter is based on the dynamic model used in the real-time simulator, because the CGF entities is different from the man-in-loop simulator, i.e. main-in-loop simulator requires high precision and perfect real-time characteristics, on the contrast, CGF requires lower precision and lower real-time characteristic. In order to simulate more than one entities in one computer, they should be built as "approximate real-time" simulators. The precision of the Runge-Kutta method is so high that the time cost is also very high. In fact, we found that the three attitudes $\mathrm{p}, \mathrm{q}$ and r are independent from each other through our experience, so the Runge-Kutta method is combined with Euler method in this paper, which can keep high precision and reduce the time cost.
Our method is: the differential equations of $u, v$ and $w$ are discretized at first, then the Euler method is used for calculating the integral, and the 4-order Runge-Kutta method [7] is used for $p, q$ and $r$.
The concrete algorithm is:

1. Access the integral step $\Delta t$;
2. Access the values of $u, v, w$ and $p, q, r$ of the last loop;
3. Calculate $\mathrm{p}, \mathrm{q}$ and r by the Runge-Kutta method;
4. Calculate $u, v$, and $w$ by the Euler method;
5. Calculate the transition matrix by the Euler method;
6. Calculate the three speed values $(u, v, w)$ and the three displacements ( $\mathrm{x}_{\mathrm{d}}, \mathrm{y}_{\mathrm{d}}$ and $\mathrm{z}_{\mathrm{d}}$ );
7. Calculate the three attitude angles $\theta, \varphi$ and $\psi$;
8. Jump to step 1 .

## 5. IMPLEMENTATION / PERFORMANCE

The above-mentioned model has been applied to a system we developed in the past, Through a series of experiments, it was tested to be able to:
i. Simulate the attitudes change of the helicopter correctly when it is flying. In the experiment, the attitudes of helicopter changes correctly according to the operator's manipulation, and it adapt to the disturb of the wind from the simulated atmosphere to keep its balance.
ii. Satisfy the real-time requirements of the system, and collaborates properly with other entities. In the experiment, based on the model, the frame rates of the system can achieve 25 to 28 frames in one minute on PIII-450M, 38 to 42 frames on $\mathrm{P} 4-800 \mathrm{M}$, about 60 frames on $\mathrm{P} 4-2 \mathrm{G}$ (all computers have been installed professional graphic cards). In contrast, when solving all equations by the Runge-Kutta method, the frame rates of the system can only achieve 7 to 10 frames in one minute on PIII-450M, 22 to 25 frames on P4-800M, less than 45 frames on P42G.
iii. Satisfy the precision requirements in rivalry training, the precision of the three attitude angle $\theta, \psi$ and $\varphi$ of the helicopter can be controlled to 0.1 degree, which make the virtual helicopter in the battle environment change its attitude very smoothly on the monitor.

## 6 CONCLUSION

Aiming at the characteristic of CGF, one type of dynamic model for helicopter has been implemented in this paper, the Runge-Kutta method is combined with the Euler method in numerical algorithm to simplify the calculation as well as not to lower the precision. At the same time, it prevents the occurrence of singularity by using quaternion.

## REFERENCES

[1] Johnson, 1997. W. "CAMRAD II, Comprehensive Analytical Model of rotorcraft Aerodynamics and dynamics" Johnson Aeronautics, Palo Alto, California.
[2] ShiChun Wang .The Dynamics of Helicopter. Beijing University of Aeronautic and Astronautic Science and Technology
[3] Yamauchi. G.K.; Heffernan, R.M.; and Gaubert, M. "Correlation of SA349/2 Helicopter Flight Test Data with a comprehensive Rotorcraft Model." European Rotorcraft Forum, Germany, September 1986; Journal of the American Helicopter Society, Volume 33, Number 2, April 1988.
[4] Chao Yang, XiaoGu Zhang. 1998. Analysis on the character of flight mechanics model of helicopter., flight mechanics.
[5] KuYu Wang. 1991. The Flight Control System of Helicopter. The LanTian Press.
[6] Harris, F.D.; Tarzanin, F.J., Jr.; and Fisher, R.K., Jr. 1970. "Rotor High Speed Performance, Theory vs. Test." Journal of the American Helicopter Society, Volume 15, Number 3.
[7] Jilin Shi, GuiZhen Liu. 1996. Numerical Algorithm. GaoJiao Press.
[8] GuoFong Pan, XinPing Zhao. 1999.The CGF in DIS . Computer Science.

## AUTHOR BIOGRAPHIES

Jianxiang Liu is associate research fellow of Beijing Institute of System Engineering , located in FengTai district, Beijing, China. His research focuses on HLA, CGF and computer distribute computing. He can be contacted by email at [13366093801@m165.com](mailto:13366093801@m165.com)

Haozhi Li is associate research fellow of Beijing Institute of System Engineering, His research focuses on Distributed Virtual Battle Field Environment and modeling, He can be contacted by e-mail at [mave_rick@21cn.com](mailto:mave_rick@21cn.com)

