TRADING VOLATILITY WITH OPTIONS ON STRADDLE

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ABSTRACT

The risk of volatility – even if many have not observed this yet – is as old as options. Together with the development of financial markets, newer and newer options, securities and forms of investments with embedded options appeared. As they spread, so grew the number of institutional and private investors running the risk of volatility.
In the eighties and nineties, markets became more and more volatile, thus the frequency of losses originating also grew. In parallel with those losses, demand to reduce the exposure to volatility also emerged. And demand creates its own supply. Banks seemed to be especially active in selling volatility products: they sold swaps and options on volatility. The risk so taken, they tried to hedge by the synthetic creation of the same volatility products. However, a good financial product not only has to be attractive for the market, but also has to be synthetically replicable.
Brenner at al [2002] tried to dissolve this contradiction. They offered investment banks an option on a combination of options (on a straddle) instead of options on volatility. This compound option becomes hedgeable without any major problem.
In this study, this new product was examined in two continuous time models. One of them is the Hull-White model [1987], which has always been considered as a benchmark in literature. In addition, in this case the correlation between the two processes – that of share price and volatility – is also easy to analyse. The other model dealt with the logarithmic mean-reverting model used by Detemple-Osakwe [2000]. This gives a good description of volatility on the stock market, and Detemple-Osakwe priced volatility options by these processes. Therefore, the results of the paper are comparable with theirs.[1]
This paper shows that although the option on volatility is an adequate asset to hedge the volatility exposure, it can have very special price changes depending on the value of the correlation between the two processes. If the correlation changes, the main characteristics of the option on straddle can change essentially. It means, that banks selling this asset and hedging synthetically will face an other source of risk: the correlation risk.
The results were obtained using the Monte-Carlo simulation method. In the preparation of the programmes and in solving simulation-related problems Gábor Benedek provided valuable help.

THE OPTIONS ON STRADDLE

Brenner, Ou and Zhang [2002] propose that the underlying of the option should not be volatility itself, but a product sensitive to it, a straddle with an ATM strike price. A great advantage of the latter is that it is traded, investors speculating on volatility have been using it for long, they know it well.
The structure of the contract is the following. The owner of the option can buy an ATM straddle in time T_1 at the maturity of the option, which expires later in time T_2. The only problem is that it is not yet known what the ATM value

[1] (Further below I refer to the Hull-White model as HW, and to the Detemple – Osakwe model as DO. By process I mean the processes driving the share price and volatility.)

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will be in time T_1. Brenner, Ou and Zhang extended their model for both the case of deterministic and stochastic volatility, from which the stochastic one is interesting for us.
Brenner, Ou and Zhang chose a mean-reverting process. Although in their model the stochastic element is a Wiener process, the size of the random element is independent form the current value of volatility.
The process followed by the underlying and that followed by volatility are the following:

$$dS_t = rS_t dt + \sigma_t S_t dZ_t^1 \qquad (1.)$$

$$d\sigma_t = \delta(\theta - \sigma_t)dt + \xi dZ_t^2 \qquad (2.)$$

where θ is the long term mean of volatility, δ is the speed of reversion to this mean, ξ is the volatility of volatility and Z^1 and Z^2 are two independent Wiener processes.
Brenner et al. showed that the value of this product is only a function of volatility and the share price. Delta hedging with the underlying of the primary option results in a volatility product depending only on volatility.

THE PROCESSES

Literature offers many kinds of procedures to model the evolution of volatility as perfectly as possible. Below we deal with two models. One of them, often treated as a benchmark in literature, is the **Hull-White model**:

$$dS = \mu S\, dt + \sigma S\, dw \qquad (17)$$

$$dV = \phi V\, dt + \xi V\, dz \qquad (18)$$

where $V = \sigma^2$, dw and dz are correlated Wiener processes, with a ρ correlation coefficient, μ is the expected return of the share, ϕ is the drift (expected growth rate) of the variance, σ is the volatility, and ξ is the volatility of volatility.
Hull and White assumed that the two processes can be correlated. The application of this model in this paper is also justified by the fact that it makes it possible to examine how correlation between the two processes effects the value of the option on straddle. Brenner, Ou and Zhang did not address this question.
The other model was taken from the article of **Detemple and Osakwe**. Detemple and Osakwe examined several processes, nevertheless they considered the mean-reverting log process as the most important. This was argumented by this model being – as they have shown – a continuous extension of the EGARCH model, hence it gives a good description of the American share prices. Accordingly, the parameters of the equations can be calculated as functions of the parameters used in the EGARCH model.

$$dS = \mu S\, dt + \sigma S\, dw$$

$$d\ln(\sigma) = (\alpha - \lambda \ln(\sigma))dt + \xi\, dz$$

Where σ is the volatility[2], dw and dz are the two ρ correlated Wiener processes, μ is the expected return of the share, α is the long-term logarithmic volatility value, λ is the speed of mean-reversion, and ξ is the volatility of volatility.
Notwithstanding the previously mentioned mean-reverting log model was analysed when pricing another volatility product by this process, that of the option on volatility. The characteristics of a **put** on straddle also were investigated.
For the sake of simplicity, it is assumed all through the analysis that the underlying is a share, which will not pay any dividend during the time examined.

ABOUT THE NUMERICAL METHODS

Form of denoting the processes

Similarly to the original articles, the logarithmic form of processes was used. This coincides with the format used by the authors in the original articles, and it ensures that the share price cannot have a negative value. The form of the two processes used in the simulations were the following:
Hull – White model:

$$S_i = S_{i-1} \cdot e^{\left[\left(r - V_{i-1}/2\right)\Delta t + u_i \sqrt{V_{i-1}\Delta t}\right]}$$

$$V_i = V_{i-1} \cdot e^{\left[\left(r - \xi^2/2\right)\Delta t + \rho\, u_i\, \xi \sqrt{\Delta t} + \sqrt{1-\rho^2}\; v_i\, \xi \sqrt{\Delta t}\right]}$$

Detemple – Osakwe model:

$$S_i = S_{i-1} \cdot e^{\left[\left(r - \sigma_i^2/2\right)\Delta t + \sigma_i u_i \sqrt{\Delta t}\right]}$$

[2] Note that here the second process is the one driving volatility and not variance as in the Hull-White model!

$$\sigma_i = \sigma_{i-1} \cdot e^{\left[(\alpha-\lambda \ln(\sigma_{i-1}))\Delta t+\xi\left(\rho\, u_i\, \sqrt{\Delta t}+\sqrt{1-\rho^2}\, v_i\, \sqrt{\Delta t}\right)\right]}$$

The description of the simulation

The length of the period was one year. Both the maturity of the option on straddle and that of the vanilla option was six months. For the sake of clarity, a year contained 360 days, which means 180 days in six months. In the simulations where the results of this paper were compared with that of Detemple and Osakwe, the maturity of the compound option was changed.

The programmes necessary for the simulations were written in Delphi programme language. The Gasdev random number generator was used, often used in economic simulations, to obtain random numbers.[3]

Each realisation was a result of a 1 000x1 000 simulation. To minimise the error in the results, each simulation was carried out ten times, and their arithmetic average was used in the analysis. As the simulation of one phase consisted of 180 periods and two random numbers were generated for one period, each compound option price was built 10x360 360 000 random numbers in order to minimise the noise of the random number generator.

Unfortunately, there is no closed analytical formula for these processes therefore the Black-Scholes setting was used for tests. The programme taking the average of ten realisations converged well to the reference values: as a result of ten realisations the difference (in absolute terms) remained between 0.1% and 0.2%.

Based on the tests, the Detemple and Osakwe model seemed to be more stabile, as the results converged faster to reference values.

Assumptions used for pricing

One of the most important questions when using stochastic volatility models is how to treat the problem of having two risk factors, share price and volatility, and not having tools to mitigate the risk of the latter. Several ideas came up to solve this problem. One of the simplest is to assume, that there is a product, the underlying of which, is volatility itself; and by using this, volatility, risk can be mitigated. In other words: the market is complete, all risk factors are traded. This assumption is used, for example by Johnson and Shanno [1987].

In this case, the objective is to price such a volatility product. Therefore to assume that such a product already exists would not be too elegant and effective. Hence the other assumption widespread in literature – and also applied among others by Hull and White – was used, according to which volatility risk can be diversified and thus volatility has no systematic risk.

One should notice that from this point on there are two correlations in the models. On the one hand, it is sometimes assumed that processes driving the share price and volatility are correlated (in certain cases they are uncorrelated to some extent), while it is always assumed that correlation between the volatility of the share price and the expected return of the market portfolio is zero. The first assumption will be a subject of the analysis while the second ensures the completeness of the market.

THE RESULTS

The values of the options of straddle

In this paper, the effects of the initial volatility, the volatility of volatility, and the correlation of the two processes driving the share price and volatility on the value of options on straddle were analysed. In most cases, the results were similar to that of vanilla options, nevertheless sometimes they were different from Brenner et al's results.

As far as the **initial volatility** is concerned, the option on straddle behaves well: the value of CoST is a positive, while that of PoST is a negative function of initial volatility for both models. But not always a convex function! In the HW model convexity is a function of the correlation coefficient. In case of negative correlation, the value of the call on straddle will be a concave function of the initial volatility.

[3] For more details on the theory and practice of simulations see Knuth [1987] and Benedek [2003].

In the DO model, puts continue to behave regularly, that is their value is a concave function of volatility. In case of the CoST the effect is very interesting. By a correlation coefficient of plus or minus one, the compound option is a concave function of volatility, while in other cases convexity was observed.

These results are not unique. Detemple and Osakwe – using the mean-reverting log process– experienced that the value of options by a high initial volatility became a concave function of volatility.

Interesting was the influence of the **volatility of volatility** as well. The volatility of volatility was examined by Brenner et al., too. In their model, the compound option behaved absolutely "regularly" as a function of the volatility of volatility. But not in these models.

In the HW model, this issue can be addressed only together with that of the correlation coefficient, thus also the next question is answered partly. In the case of zero correlation both CoST and PoST behave regularly, while in case of strong negative correlation the volatility of volatility can have a reducing effect on the value of options in certain phases. The issue is even more important as in practice volatility and share price are typically negatively correlated.

Unique characteristics can also be found in the DO model. The atypical behaviour of option is not only a function of correlation. In case there was zero correlation between the two processes, the CoST did behave as a vanilla option. But not in the case of put options. By higher exercise prices the value of the PoST was a negative function of the volatility of volatility! At the same time by reducing the exercise price the order was restored.

The question is raised whether this relation changes by changing the correlation coefficient. Therefore, the analysis of the value of the PoST should be continued. The value of PoST was analysed by correlation coefficients of minus and plus one.

It can be easily observed that the behaviour observed is independent from the correlation between the two processes, the sensitivity of the option's price to the volatility of volatility will be a function of the exercise price. By a high exercise price, in case of strongly ITM put options, the volatility of volatility reduces the price of the compound option, while in case of OTM put options, it is going to increase the same.

There was no such problem in case of call on straddle, they behaved as an option on volatility independent from the correlation coefficient.

In the end, it is worth to note, that all throughout the analysis of Brenner et al., the compound option behaved regularly, it was a positive function of the volatility of volatility. Of course, it should be mentioned that they only dealt with call options, in this sense my results do not differ from theirs.

The effects of the **correlation** between share price and volatility processes were also analysed. The issue of correlation was important as it was not addressed by Brenner et al., whereas it is a general observation on the market that there is negative correlation between the volatility and the share price.

In the case of the HW model, the results were interesting. The hint can be put this way: in case of close-to-ATM options, the increase of the correlation coefficient increases the value of both CoST and PoST. As in general, ATM options are the most frequently traded on the market, this rule is going to be the most important one for banks selling these types of combined derivatives. However, it is good to know that when trading with an exercise price different from ATM, the effect might change. Of course in practice, it is also an important question, whether the HW model gives a good estimation of reality, the evolution of the share price and volatility.

In case of the DO model the value of CoST is more sensitive to changes of correlation, than that of PoST.[4] The value of the compound options – both calls and puts – changes parabolically as a function of the correlation between the two processes. While in case of CoST, the price is the highest by zero correlation, in case of PoST is the value of the compound option the lowest.

By CoST, regardless of whether we differ positively or negatively from zero correlation, the value of the option reduces. By PoST, the effect is the other way round. Zero correlation

[4] The strength of the effect also depends on the volatility of volatility.

gives the lowest option price. As the absolute value of ρ grows, PoST becomes more valuable.
The importance of the results obtained can again be supported by practice. The value of correlation between share price and volatility can change, therefore the option might be mispriced. When disregarding correlation, call options are systematically overpriced and puts are systematically underpriced.
Changing the exercise price did not cause such a difference as in the HW model. The influence is mostly the same by all exercise prices.
The effects of the correlation between the two processes and of the volatility of volatility are interdependent, the simultaneous increase of the two factors will increase the value of CoST and reduce the value of PoST. In case the latter is zero, the correlation between the two processes ceases to exist. Accordingly the higher the volatility of volatility, the stronger effect will correlation have.

CONCLUSION

To sum it up, calls on straddle can have special characteristics by other processes than that analysed by Brenner et al.. They not always behave regularly. As it was shown, the correlation of the two processes has a significant effect on the value of the compound options. As a consequence, if we create a volatility neutral position by using options on straddles, we will have a correlation sensitive position. So we change one source of risk for another one.
On the other hand, the value of puts on straddle was analysed as well. As can be seen, calls and puts can work differently in certain situations. In some cases puts can be more attractive products for hedgers than their call counterparties.

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