SIMULATION OF TURNING RATES IN TRAFFIC SYSTEMS

Balázs KULCSÁR
István VARGA

Department of Transport Automation,
Budapest University of Technology and Economics
Budapest, H-1111, Bertalan L. u. 2., Hungary
e-mail: kulcsar@kaut.kka.bme.hu, ivarga@sztaki.hu

KEYWORDS

traffic simulation, split rate estimation, Kalman-filter, Moving Horizon Estimation

ABSTRACT

Certain variables of dynamic system can not be measured, however they play an important role for control system strategies. In such a situation the approximation, computer based simulation of these variables could be useful for further techniques. There are some estimation methods which can determine the traffic flow in a traffic network. Based on the knowledge of these values, the simulation of the turning rates can be made. The paper treats the simulation of split rates in the traffic systems modelled in terms of linear time varying system using different filtering approaches.

The paper proposes three methods for simulation of turning rates in a basic traffic network. First the unconstrained Kalman filtering, and secondly the algorithm that has been developed for traffic systems is based on the unconstrained and constrained Moving Horizon Estimation(MHE) is presented. The constrained MHE problem for traffic systems is modelled in terms of linear time varying system and solves the split rate estimation process. The estimation is subjected to equality and inequality constraints. A numerical example is solved to demonstrate the Moving Horizon Estimation of split variables.

INTRODUCTION

Simulation is a reliable tool that one uses for model based design techniques of a real or an abstract system and to conduct experiments with, in order to understand the behavior and to evaluate various theoretic strategies for operating the system.

Applications of traffic simulation can be classified in several cases. Some basic classifications are the division between microscopic, mesoscopic and macroscopic, and between continuous and discrete time approach. According to the problem area we can separate intersection, road section and network simulations. Special fields are traffic safety and the effects of advanced traffic information and control systems.

A newly emerged area is the demand estimation through microscopic simulation. The dynamic aspect of traffic simulation in a traffic system needs the previously measured or estimated volumes of vehicles. Though the measurement of certain variables in the dynamic description are rather costly, one tries to estimate them. The observation of permanently varying turning rates, in a simple intersection, are rather costly, however the amount of the turning vehicle could be applied for traffic light harmonization, generally speaking for control.

One divides the intersection into three parts such as entry, exit and internal flows. The measurement of both the entry and the exit flows might be assumed. Traffic density cannot be measured without error, so the idealized flow plays role only in theoretical aspects. A model setup of entry-exit travel demands regarding an intersection allows estimation methods to determine the internal link flows. The key of the model buildup is the split parameter ratios. The spilt rate determines the turning percentage of the vehicles entering a traffic system. If one assumes that these turning rates are slowly varying split probabilities, the methods to determine probabilities are called split ratio methods ([4],[10]). The split rates define a turning proportion. The stochastic view on creating the model was elaborated in [11].

There exist many estimation techniques, for giving reliable estimation on dynamic OD matrix, their results, however, could be different. The short review on OD estimation begins with the Least Squares (constrained or not), statically based methods such as Likelihood methods([11]) and Kalman filtering ([2],[3]), or Bayesian estimator ([20]). Sometimes, combined estimators, using constraints or apriori knowledge about the intersection can be applied.

Constraints must be taken into account in course of dynamic OD estimation. A class of optimal state estimation methods are called Moving Horizon Estimation (MHE) methods([6],[13],[16]). The MHE can be concerned as the dual of the Model Predictive Control, though some special assumptions must be given for filter stability. Another advantage of Moving Horizon Estimation can be

Proceedings 16th European Simulation Symposium
György Lipovszki, István Molnár © SCS Press, 2004
ISBN 1-56555-286-5(book) / ISBN 1-84233-106-x(CD)

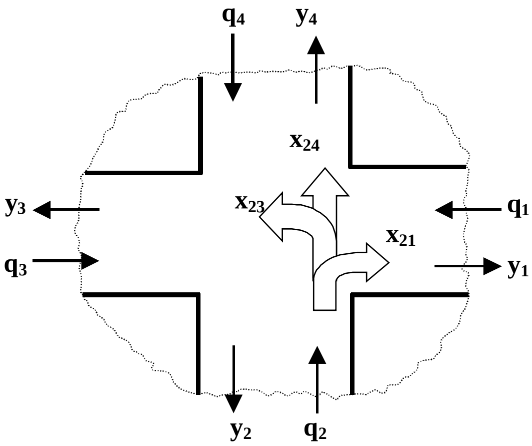

Figure 1: A simple intersection

the fact that constraints assumption can be combined in the estimation process. In the following space the Moving Horizon state estimation method is applied in intersection model.

The paper is divided into 5 chapters. After a short introduction, the problem is setup in the first section. The second section briefly summarizes the estimation techniques for split rate approximation and shows how to apply them for a basic traffic system. The third part gives a numerical example. The conclusion contains further research problems.

PROBLEM STATEMENT

One of the basic elements in traffic network systems is the intersection. A basic intersection is given in Figure 1. Let us denote the volumes occurring in a simple intersection.

To show the problem the following variables are defined:

- $q_i(k)$ the traffic volume (the number of vehicles) entering the intersection from entrance i, during time interval $k = 1,2,..N$
- $y_j(k)$ the traffic volume (the number of vehicles) leaving the intersection from exit j, during time interval $k = 1,2,..N$
- $x_{ij}(k)$ the percentage of $q_i(k)$ (turning rate) that is destinated to exit j, $k = 1,2,..N$.

At the intersection there are no traffic light and the right of way is not regularized, since from point of view estimation it does not take into account, only for control purpose has importance.

Let us consider the following intersection model

$$y_j(k) = \sum_{i=1}^{m} q_i(k)x_{ij}(k) + v_j(k), \tag{1}$$

where $i = 1,\ldots,n$ and $j = 1,\ldots,m$. $v_j(k)$ is a zero mean noise term. The input measurement is a noisy term, since $q_i(k) = \tilde{q}_i(k) + \zeta_i(k)$, with the same assumption for the noise $\zeta_i(k)$ as above.

Split variables are independent trials. The model and its constraints are given by

$$x_{ij}(k+1) = x_{ij}(k) + w_{ij} \tag{2}$$
$$1 \geq x_{ij} \geq 0 \tag{3}$$
$$\sum_{j=1}^{m} x_{ij}(k) = 1. \tag{4}$$

The random variation in split parameter is small, and the $w_{ij}(k)$ is a zero mean random component. All random components ζ, v, w are mutually independent terms.

For the sake of simplicity, let us arrange all elements of the OD matrix in a single vector and use the following notations:

$$x_k = [x_{ij}(k)]^T$$
$$w_k = [w_{ij}(k)]^T$$
$$v_k = [v_j(k)]^T$$

The problem is to observe the x_{ij} states under certain condition. The latest estimation of the split probabilities can be treated as a filtering problem. In the following section one tries to emphasize the effectiveness of the constraint Moving Horizon Estimation (MHE) method as a reliable state observer of split ratios. To understand the difference between them Kalman filtering and unconstrained MHE is computed and simulated as well.

TURNING RATE ESTIMATION METHODS

Usually for real systems the states can not be measured since the output map is only a subset of the whole state space. In many application the knowledge of the states has particular importance for implementation, for state feedback control problems. State estimation for control purpose becomes primordial and many estimation technique has been developed already.

State estimation gives us the possibility to observe via output the unmeasured states, and for stochastic systems to reduce the state and measurement noise occurring as well. Though for linear stochastic system different estimation methods provide quite a good approximation of the real state, but the realization of nonlinear state estimation under stochastic noise causes problems.

In our case the turning rate estimation of a simple intersection has many control objectives, i.e. the turning amount of the vehicle in a direction could modify the optimal traffic light control. Certain class of noisy linear discrete time system can be described is as follows:

$$x_{k+1} = A_k x_k + B_k u_k + G_k w_k \tag{5}$$
$$y_k = C_k x_k + D_k u_k + v_k \tag{6}$$

One can neglect the control input, since the split rate dynamic can be assumed as a

$$x_{k+1} = Ax_k + Gw_k \tag{7}$$
$$y_k = C_k x_k + v_k \tag{8}$$

with x_0 given and with $G_k = A_k = I_n$.

where A_k shows the propagation of the states from x_k to x_{k+1}, the control input u_k affects the dynamic system through B_k input direction map, and $G_k w_k$ is the weighted state noise with zero mean random signal w_k. G_k is called noise distribution matrix and colors the white noise w_k. Usually noisy systems are described with an additional disturbance term in dynamic equation.

The output map C_k can be a time or parameter varying map deciding about measured outputs, under output noise. When stochastic noise are presents, such as w_k, respectively v_k, the resulting state estimator one is often called filter. Although it provides optimal estimation under the noisy, measured output information.

Stability of filter can be shown by computing the error system for nominal case, i.e. the simulation of noiseless filter and real system from a different initial condition. The filter design methods needs the probabilistic description of the noisy term, such as probability density function. As an example of Kalman filter, which gives optimal solution with minimizing covariance of state errors.

Stochastic programming framework for estimation exists as well. Stochastic optimization methods consider zero mean random noise with Gaussian distribution.

When comparing optimal filter design methods, one usually starts with general Least Square(LS) method, which has been first presented in the second parts of 60's. Since computer based numerical solution was unable to gain the estimation results, practical implementation was impossible at the time. The reason why nowadays it has been so successful is the possibility of including explicit information about estimation processes. The implementation of limitation concerning state or measurement noise can be understood from the Nature, since unlimited disturbance can not be interpreted.

General outlook of stochastic state estimation process can be seen on Figure 2.

From point of view numerical realization of state estimation recursive algorithm formulation is necessary. The most general formulation of LS for dynamic systems is the Batch or Full Information Estimation (BE,FIE). It offers the possibility to maintain equality or inequality constraints in an infinite horizon. The main drawback of this solution of state estimation is the computational requirements, because when applying, the entire past behavior of estimated process is familiar for estimation.

The above estimation problem becomes a time variant case for (8).

The batch estimator is given by

$$\min_{(\bar{x}_0, \hat{w}_{-1|k}, \ldots, \hat{w}_{k-1|k})} \Psi_k$$

$$\Psi_k = \hat{w}_{-1|k}^T Q_0^{-1} \hat{w}_{-1|k} + \\ + \sum_{j=0}^{k-1} \hat{w}_{j|k}^T Q^{-1} \hat{w}_{j|k} + \\ + \sum_{j=0}^{k} \hat{v}_{j|k}^T R^{-1} \hat{v}_{j|k},$$

subject to:

$$\hat{x}_{0|k} = \bar{x}_0 + \hat{w}_{-1|k}$$
$$\hat{x}_{j+1|k} = A\hat{x}_{j|k} + G\hat{w}_{j|k}$$
$$y_j = C\hat{x}_{j|k} + \hat{v}_{j|k}$$

with R^{-1}, Q^{-1} which are symmetric positive semi-definite noise weighting matrices. While Q_0 penalizes the $\bar{x}_{k-N}$ initial state, R^{-1} weights the output prediction error and Q^{-1} penalizes all estimated state noise.

The optimization problem grows step-by-step and becomes intractable even for small dynamic systems.

The general stability property of unconstrained batch estimation can be applied for constrained with some special assumptions. The feasibility of the constrained optimization comes, over all, from the initial values of the estimated state and noise. Naturally, for constrained batch an unstable A could cause unstable estimator, since the unstable true system trajectories cannot be observed with constraints on estimator states. However, general (nominal) asymptotic stability even for unstable A can be ensured.

The Moving Horizon Estimation scheme can be seen in Figure 3.

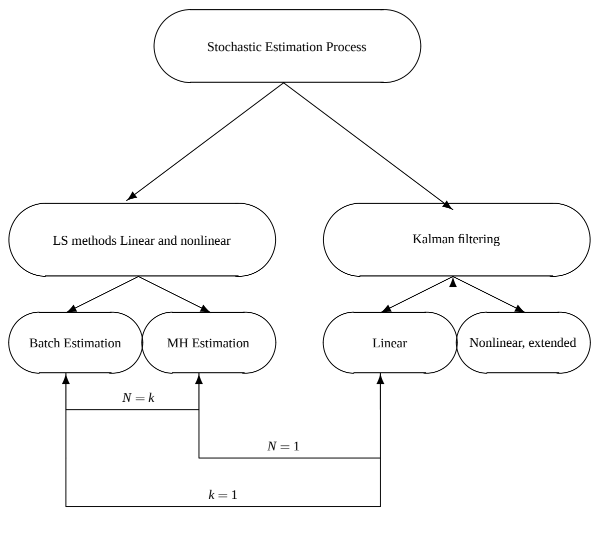

Figure 2: Stochastic Estimation Process

Let the generalized MHE optimization criteria be defined by the following functional

$$\min_{(\bar{x}_0,\hat{w}_{k-N-1|k},\ldots,\hat{w}_{k-1|k})} \Psi_k$$

$$\begin{aligned}\Psi_k = {} & \hat{w}^T_{k-N-1|k} Q_N^{-1} \hat{w}_{-1|k} + \\ & + \sum_{j=k-N}^{k-1} \hat{w}^T_{j|k} Q^{-1} \hat{w}_{j|k} + \\ & + \sum_{j=k-N}^{k} \hat{v}^T_{j|k} R^{-1} \hat{v}_{j|k} + \Psi^*_{k-N},\end{aligned}$$

subject to:

$$\begin{aligned}\hat{x}_{k-N|k} &= \bar{x}_{k-N} + \hat{w}_{k-N-1|k} \\ \hat{x}_{j+1|k} &= A\hat{x}_{j|k} + G\hat{w}_{j|k} \;\; j = k-N-1,\ldots,k-1 \\ y_j &= C\hat{x}_{j|k} + \hat{v}_{j|k} \quad j = k-N-1,\ldots,k\end{aligned}$$

If the expected output is small, R^{-1} has to be chosen large, compared to Q^{-1}, and the resulting sensor noise vector becomes small, compared to $\hat{w}_{j|k}$. On the other hand, if our measurements are not reliable, Q^{-1} should be chosen large, compared to R^{-1}.

Ψ^*_{k-N} is the so-called arrival cost, which is analogue to the *cost to go* in MPC technique. The arrival cost summarizes all knowledge about the best estimation before the N-th step. For unconstrained linear case, the arrival cost can be expressed explicitly. If state or noise inequality constraints, or nonlinearities are present, we have no analytic expression to generate the arrival cost. Though analytic approach is unavailable, an *approximate* cost may be given. When inequality constraints are inactive, the approximation is exact. Therefore, the poor choice of the arrival cost leads to the filter's instability. To find the initial condition of General MHE, one uses a batch estimation for the first $N-1$ step estimates.

To slide between windows the filtered estimate update is preferred.

Another possibility of state estimation is the use of Kalman filter. The Kalman filter could be applied widely in traffic systems. This has been published in numerous papers ([4],[**?**]). This method, based on Gaussian distributions of random variables, is defined on a probability framework of the unknown split parameters. Kalman filter equations ([8]) can be formulated as recursive ones started with an initial condition. The optimal estimation depends on the choice of state noise covariance (Q) and on the output noise covariance (R) weights. The Kalman estimator can be applied subject to inequality constraints by using stochastic programming([19]). The connection between Kalman filtering and full information estimation is known.

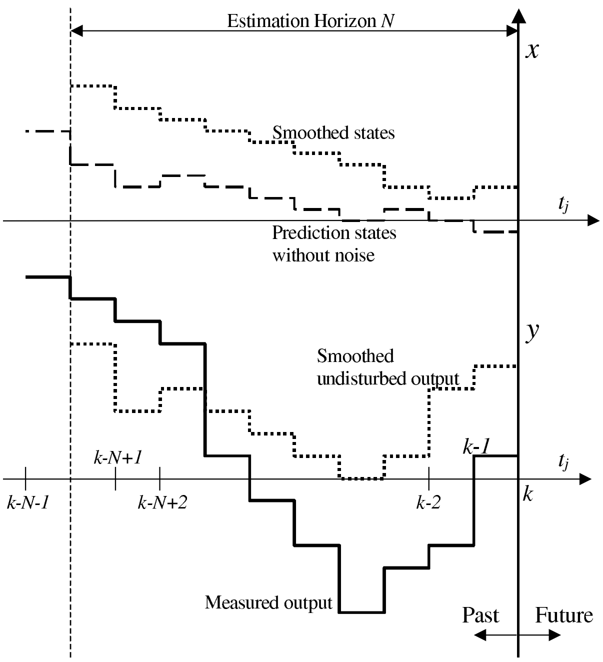

Figure 3: General Moving Horizon Estimation process

EXAMPLE

To show the difference between estimation techniques let us consider the following solution of estimation of split ratio. The solution is based on standard Kalman filtering approach, on general unconstrainted MH Estimation, and constrained MH Estimation with the equality constraints (4) and dynamic inequality constraint.

One returns to the intersection model which is now given by:

$$\begin{aligned}x_{k+1} &= x_k + u_k + w_k \\ y_k &= C_k x_k + v_k,\end{aligned}$$

where C_k contains the elements of q_i, a time varying output map. The structure of C_k depends upon the layout of the intersection.

$$C_k = \begin{bmatrix} q_1 & 0 & q_2 & 0 \\ 0 & q_1 & 0 & q_2 \end{bmatrix}$$
$$q_{k+1} = \widetilde{q}_k + \zeta_k$$

and where q_k is the noisy input volume of the vehicles entering the intersection, with the sensor noise ζ_k a zero mean random signal with the appropriate dimension.

The number of split parameter needs to be well chosen in our situation the $x = [x_{13} \quad x_{14} \quad x_{23} \quad x_{24}]^T$, the number of outputs $y_k = [y_3 \quad y_4]^T$. One should denote that in split parameters two types of variation are present: the permanent systematic component, with a zero average during a time period, and the the random component which is assumed to be small respect to 1.

Let us suppose to have 1 sample in every second. Let the horizon comprise 1 sample, and by applying diagonals R, Q and Q_0, the following results are gained. The simulation covered take 1 hour, which can be seen on the Figure 4.

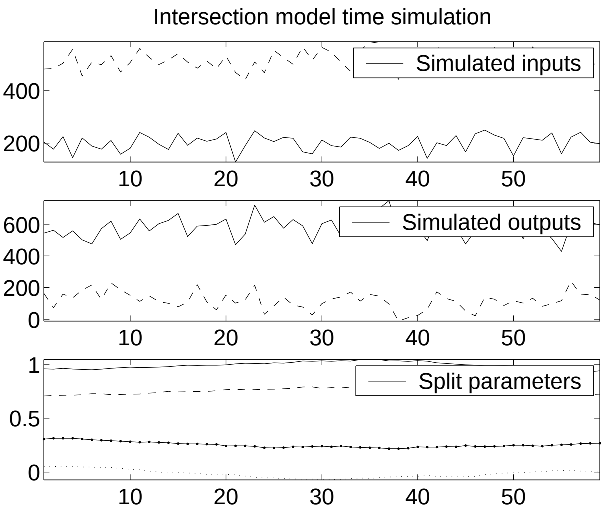

Figure 4: Variables of the basic intersection in time domain

For numerical computation and simulation of the estimated split states one uses quadratic programming either for unconstrained or for constrained MH. The Kalman filtered states are computed by a recursive algorithm.

For solving MHE numerically, either one may use a recursive algorithm, which can be derived from Lagrange multiplier method([6]), or quadratic programming for unconstrained, respectively for constrained MH. Simulating the split parameter for intersection (Figure 1.), the result can be seen in Figure 5.

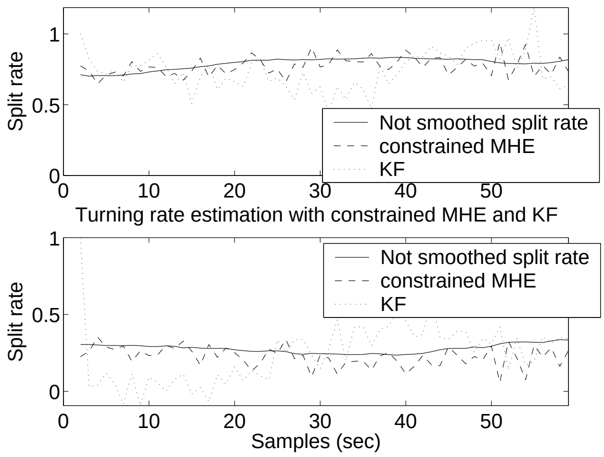

Figure 5: x_{12}, x_{23}, x_{32} split parameter time behaviour

In the Figure 5 only the Kalman filter and the constrained MHE is plotted. Since for $N = 1$ the unconstrained MHE and the Kalman filtering problem gives almost the same result (numerical computational error). The only difference is the estimation idea, because while for Kalman filtering the prediction is always a forward, the unconstrained MH approach is defied as backward calculation.

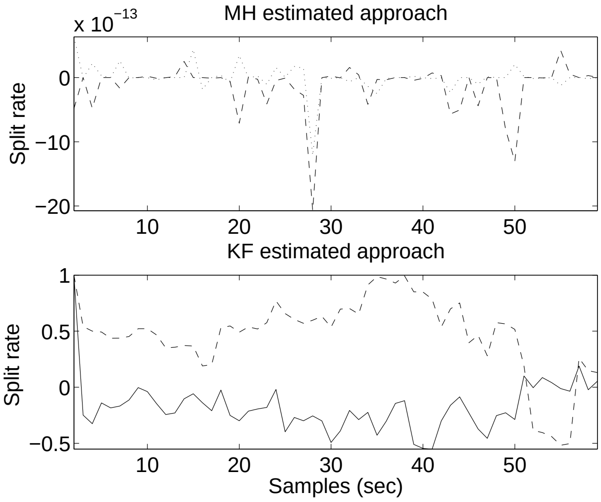

Figure 6: Equality constraints $x_{13} + x_{14} - 1 = 0$ and $x_{23} + x_{24} - 1 = 0$ time behaviour

As it has been shown in Figure 6, the simulated turning rates (supposed to be real), are estimated with the MHE process.

CONCLUSION

The article summarizes the Moving Horizon Estimation approach for a simple traffic system, an intersection. In traffic engineering the estimation of split variables is important for further optimal traffic light control strategies.

The MHE optimal estimation method shows a possible way for including constraints into the design procedure. One could possibly extend the state estimation, based on MHE algorithm with some additional constraints in inequality form on states, noise or other variables. The selection of weighting matrices and estimation horizon and the good approximation of arrival cost influence the performance of the estimation.

A numerical example has been shown to demonstrate how to apply the Moving Horizon technique for split rate observation.

The general MHE technique could be applied to nonlinear processes which will be in the focus of our traffic system estimation research.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the contribution of Hungarian National Science Foundation (OTKA, Grant number T046220)

References

[1] Bemporad, A.,D. Mignone, M. Morari "Moving Horizon Estimation for Hybrid Systems and Fault Detection" *ACC San Diego, California, 1999*

[2] Cremer, M. and Keller, H. "Dynamic Identification of Flows from Traffic Counts at Complex Intersections" *Proc. 8th Int. Symposium on Transportation and Traffic Theory, University of Toronto Press, Toronto Canada, 1981, pp 199-209.*

[3] Cremer, M. "Determining the Time-Dependent Trip Distribution in a Complex Intersection for Traffic Responsive Control" *IFAC Control in Transportation Systems, Baden-Baden, Germany 1983, pp. 214-222*.

[4] Cremer, M. and Keller, H. "A New Class of Dynamic Methods for the Identification of Origin-Destination Flows" *Transportation Research-B, 1987, Vol. 21B, No. 2, pp. 117-132*.

[5] Douglas, G. R., J. H. Lee, "On the use of constraints in least squares estimation and control" *Automatica, 2002, Vol 38 1113-1123*

[6] Findeisen, P. K. "Moving Horizon State Estimation of Discrete Time Systems" *MS Thesis, U. of Wisconsin-Madison*.

[7] Hannah Michalska and David Q, Mayne "Moving Horizon Observers and Observer-Based" *IEEE Transactions on Automatic Control, Vol. 40, No. 6, 1995, pp. 995-1006.*

[8] Kalman, R.E. "A New Approach to Linear Filtering and Prediction" *Journal of Basic Engineering (ASME), 1960, 82D, pp. 35-45*.

[9] Kenneth R. Muske and James B. Rawlings, Jay H. Lee, "Receding Horizon Recursive State Estimation" *American Control Conferences, 1993, Vol 1. pp. 900-904.*

[10] Nihan, N.L. and Davis, G.A. "Recursive Estimation of Origin-Destination Matrices from Input/Output Counts" *Transportation Research B, 1987, Vol. 21B, No. 2, pp. 149-163*.

[11] Nihan, N.L. and Davis, G.A. "Application of Prediction-Error Minimization and Maximum Likelihood to Estimate Intersection O-D Matrices from Traffic Counts" *Transportation Science, 1989, Vol 23, No. 2*.

[12] Papageorgiou, M. "Concise Encyclopedia of Traffic and Transportation Systems" *Pergamon Press, 1991*.

[13] Rao,V. C. "Moving Horizon strategies for the constrained Monitoring and Control of Nonlinear Discrete-Time Systems" *PhD Thesis U. of Wisconsin-Maidson, 2000*

[14] Rao,V. C., J. B. Rawlings, "Constrained Process Monitoring: Moving-Horizon Approach" *AIChE journal, 2002 , Vol 48. pp. 97-109.*

[15] Rao,V. C., J. B. Rawlings,D. Q. Mayne, "Constrained State Estimation for Nonlinear Discrete-Time Systems: Stability and Moving Horizon Approximations" *IEEE Transaction on Automatic Control, 2003 , Vol 48. pp. 246-258.*

[16] Tyler, M., M. Morari, "Stability of Constrained Moving Horizon Estimation Schemes" *Automatica, 1996, Vol 17. pp. 1410-1425.*

[17] Tyler, M., K. Asano,M. Morari, "Application of Moving Horizon Estimation Based Fault Detection to Cold Tandem Steel Mill" *ETH Technical Report AUT96-06*

[18] Zijpp, N.J. Van Der,R. Hamerslag, "An Improved Kalman Filtering Approach to Estimate Origin - Destination Matrices for Freeway Corridors" *Transportation Research Records, 1994, No.1443, pp.54-64.*

[19] Zijpp, N.J. van der "Dynamic origin-destination matrix estimation from traffic counts and automated vehicle identification data" *Transportation Research Record No. 1607. TRB, Washington, DC, 1997, pp. 87-94.*

[20] Zijpp, N.J. van der "Comparison of methods for dynamic origin-destination matrix estimation" *IFAC Transportation Systems Symposium, Chania, Grecce, 1997, pp. 1445-1450.*

AUTHOR BIOGRAPHIES

Balázs KULCSÁR was born in Budapest, Hungary. After graduating as traffic engineer in 1999 at the Budapest University of Technology and Economics, he started his PhD study at the Department of Transport Automation. Presently he works as research assistant at the same department in half time. He is working for the Computer and Automation Research Institute of the Hungarian Academy of Sciences. Any complementary information can be found on http://www.kka.bme.hu/oktkut/munkatarsak.htm.

István VARGA was born in Budapest, Hungary. He graduated in traffic engineering in 1998 at Budapest University of Technology and Economics (BUTE). Presently, he is doing PhD studies at the same university. Further information can be found at www.sztaki.hu/scl